North MaharashtraUniversity ; Jalgaon.

Question Bank

S.Y.B.Sc. Mathematics (Sem –II)

MTH – 221 . Functions of a Complex Variable.

Authors ;

Prof. M.D.Suryawanshi (Co-ordinator)
Head, Department of Mathematics,
S.S.V.P.S.'s L.K.Dr.P.R.Ghogrey Science College , Dhule .

Prof. U.S.Shirole
Department of Mathematics,
S.S.V.P.S.'s L.K.Dr.P.R.Ghogrey Science College , Dhule.

Prof. A.G.Patil
Kisan Mahavidyalaya, Parola, Dist.- Jalgaon.

Prof. J.G.Patil
S.S.V.P.S.'s Arts , Commerce and Science College, Sindkheda
Dist. Dhule.

Unit – 1 :

Functions of a Complex Variable.

I) Questions of Two marks :

1) The $\lim\limits_{z\to 2+3i} \left[3x+i(2x-4y)\right]$ is - - - - - -

2) Does $\lim\limits_{z\to 0} \dfrac{\bar{z}}{z}$ exist

3) What are the points of discontinuties of f(z) = $\dfrac{2z-3}{z^2-2z+2}$

4) Write the real and imaginary parts of f(z) = z^3 where z = x+iy.

5) Find the limit , $\lim\limits_{z\to 1-i} \left[x+i(2x+y)\right]$

1) Does Continuity at a point imply differentiability there at. Justify by an example.
2) Define an analytic function .
3) Define singular points of an analytic function f(z).
4) Find the singular points for the function f(z) =$\dfrac{z-2}{(z+1)(z^2+1)}$
5) Define a Laplace's Didifferential Equation for $\Phi(x,y)$.
6) What is harmonic function ?
7) What do you mean by f(z) is differentiable at ?
8) Is the function u = $\dfrac{1}{2}.\log\left(x^2+y^2\right)$ harmonic?
9) When do you say f(z) tends to a limit as z tends to z_0?
10) State Cauchy- Riemann equations.
11) State the necessary condition for the function f(x) to be abalytic.
12) Every differential function is continuous . True or False.

II) Multiple Choice Questions :

1) If $\lim\limits_{z\to 1-i}$ [x+i(2x+y)] = p+iq , then (p,q) = - - - - - .

(i) (1,1) (ii) (-1,1) (iii) (1,-1) (iv) (-1,-1)

2) The function f(z) = $\dfrac{xy}{x^2+y^2}$ when z$\neq$0 and f(0) =0 is

(i) Continuous at z = 0 , (ii) Discontinuous at z =0
(iii) Not predictable , (iv) Constant

3)A Continous function is ifferential :

(i) True ,(ii) False.
(iii)True & False, (v) True or False

1) A function $\Phi(x,y)$ satisfying Laplace equation is called

(i) Analytic (ii) Holomorphic
(iii) Harmonic, (iv)Non-hormonic

2) Afunction f(z) = e^z is

(i) Analytic everywhere , (ii) Analytic nowhere
(iii) only differentiable, (iv) None

3) If f(z) = u – iv is analytic in the z-plane , then the C-R equations satisfied by it's real and imaginary parts are ,

(i) $u_x = u_y$; $u_y = -v_x$ (ii) $u_x = -v_y , u_y = v_x$

7) An analytic function with constant modulus is

(a) Constant , (b) not constant , (c) analytic , (d) None of these.

8) A Milne – Thomson method is used to construct

a) analytic function , b) Continuous function

c) differentiable function, d) None of these.

III) Questions for Four marks ;

1) Define he continuity of f(z) at z = z_0 and examine for continuity at z=0 the function

$$f(z) = \frac{x^4 y(iy-x)}{(x^8+y^2)\bar{z}} ; z \neq 0$$

1) Define limit of a function f(Z).

Evaluate ; $\lim\limits_{z \to \frac{2\Pi i}{3}} \frac{z^3+8}{z^4+4z^2+16}$

2) Prove that a differentiable function is always continuous . Is the converse true ? Justify by an example.

3) Use the definition of limit to prove that, $\lim\limits_{z \to 1-i} [x+i(2x+y)] = 1+i$

5) Show that if $\lim\limits_{z \to z_0} f(z)$ exists, it is unique

6) Prove that $\lim\limits_{z \to 0} \frac{\bar{z}}{z}$ does not exist

4) Prove that $\lim\limits_{z \to 0} \frac{x^3 y(y-ix)}{(x^6+y^2).z}$ does not exist , where $z \neq 0$

5) Evaluate : $\lim\limits_{z \to i} \frac{z^5 - i}{z+i}$

6) 9) Evaluate : $\lim\limits_{z \to 1+i} \frac{z^4+4}{z-1-i}$

10) Examine for continuity the function , f(z) = $\frac{z^4+4}{z-2i}$ at $z \neq 2i$

= 3+4i ; z=2i, at z = 2i

11)If f(z) = $\frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ when $z \neq i$ and f(i) = 2+3i , examine f(z) for continuity At z =i..

12)Show that the function f(z) = $\bar{z}$ is continuous everywhere but not differentiable .

13) Define an analytic function . Give two examples of an analytic function.

14) Show that f(z) =$|z|^2$ is not analytic at any point in the z-plane .

15) State and prove the necessary condition for the f(z) to be analytic . Are these conditions sufficient ?

16) State and prove the sufficient conditions for the function f(z) to be analytic.

17) Prove that a necessary condition for a complex function w = f(z) = u(x,y)+iv(x,y) to be analytic at a point z =x+iy of its domain D is that at (x,y) the first order partial derivatives of u and v with respect to x and y exist and satisfy the Cauchy – Riemann equations : $u_x = v_y$ and $u_y = -v_x$.

18) Prove that for the function F(z) = U(x,y) + V(x, y), if the four partial derivatives U_x, U_y, $V_x$and V_y exist and are continuous at a point z = x + iy in the domain D and that they satisfy Cauchy-Riemann equartions: $U_x = V_y$; $U_y = -V_x$ at (x, y), then F(z) is analytic at the point z = x + iy.

19) Show that the function defined by $F(z) = \sqrt{|xy|}$, when $z \neq 0$ and F(0) = 0, is not analytic at z = 0 even though the C-R equations are satisfied at z = 0.

20) Define $F(z) = z^5 |z|^{-4}$; if $z \neq 0$
$= 0$; if z = 0.
Show that F(z) is not analytic at the origin even though it satisfies C-R equations at the origin.

21) Show that the function $F(z) = \dfrac{x^3(1+i)) - y^3(1-i)}{x^2+y^2}$ when $z \neq 0$ and F(0) = 0 is continuous at z = 0 and C-R equations are satisfied at the origin.

22) If F(z) and $\overline{F(z)}$ are analytic functions of z, show that F(z) is a constant function.

23) If F(z) is an analytic function with constant modulus, then prove that F(z) is a constant function.

24) Show that $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} = 4\dfrac{\partial^2}{\partial z \partial \bar{z}}$.

25) Show that $F(z) = e^{\bar{z}}$ is not analytic for any z.

26) Show that $\lim\limits_{z \to 0} \dfrac{x^2 y}{x^4 + y^2}$ does not exist.

27) Show that if W = F(z) = 3x - 2iy, then $\dfrac{dW}{dz}$ does not exist.

28) Show that the function $F(z) = \dfrac{x^3(1+i)) - y^3(1-i)}{x^2+y^2}$ when $z \neq 0$ and F(0) = 0 is continuous and that C-R equations are satisfied at the origin but F'(0) does not exist .

29) Show that the function defined by $F(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}$; $z \neq 0$
$= 0$; z = 0.
satisfies C.R. equations at z = 0 but not analytic there at.

30) If F(z) is an analytic function of z then show that

(i) $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}|F(z)|^2 = 4|F'(z)|^2$

(ii) $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}[RF(z)]^2 = 2|F'(z)|^2$

UNIT : 2 Laplace Equation and Complex Integration

I Questions of TWO marks

1) Define Laplace Differential equation.

2) Define harmonic and conjugate harmonic functions.

3) True or False:

i) If F(z) is an analytic function of z, then F(z) depends on $\bar{z}$.

ii) If F(z) and $\overline{F(z)}$ are analytic functions of z, then F(z) is a constant.

iii) An analytic function with constant modulus is constant.

4) Is u = x^2 - y^2 a harmonic function? Justify.

5) Show that v(x, y) = x^2 - y^2 + x is harmonic function.

6) Show that u(x, y) = $e^{-y}\sin x$ is a harmonic function.

7) Prove or disprove: u = $y^3 - 3x^2y$ is a harmonic function.

8) Show that v = x^3 - $3xy^2$ satisfies Laplace's differential equation.

9) State Cauchy-Goursat Theorem.

10) Define simple closed curve.

11) Define the term Simply connected region.

12) Define Jordan Curve.

13) State Jordan Curve theorem.

14) Evaluate $\int_C \frac{1}{z-a}dz$ where C is circle $|z-a| = 2$.

15) Evaluate $\int_0^{3+i} z^2 dz$ along the line x = 3y.

II Multiple Choice questions1 mark each

1) The harmonic conjugate of $e^x\cos y$ is …………….
(a) $e^x\cos y + c$ (b) $e^x\sin y + c$ (c) $e^x + c$ (d) None of these

2) The harmonic conjugate of $e^{-y}\sin x$ is …………….
(a) $e^{-y}\cos x + c$ (b) $e^{-y}\sin x + c$ (c) $e^{-x}\cos y$ (d) None of these

3) The value of the integral $\int_C (12z^2 - 4/z)\, dz$ where C is the curve y = $x^3 - 3x^2 + 4x - 1$ joining points (1,1) and (2,3) is given by ……………….
(a) -156 + 58i (b) -156 - 58i (c) 50 (d) None of these

4) The value of $\int_0^1 z\, e^{2z}\, dz$ will be
(a) e (b) 1 / 4 (e^2 +1) (c) 1 / 4 (e^2 -1) (d) None of these

5) The value of the integral of 1 / z along a semicircular arc from -1 to 1 in the clockwise direction will be ……………….
(a) zero (b) $-\pi i$ (c) πi (d) None of these

Questions of THREE marks

1) If $F(z) = u + iv$ is an analytic function then show that u and v both satisfy Laplace's differential equation.

2) If $F(z) = u(x,y) + iv(x, y)$ is an analytic function, show that $F(z)$ is independent of $\overline{z}$.

3) Explain the Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ when the real part u is given.

4) Explain the Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ when the imaginary part v is given.

5) Find an analytic function $F(z) = u + iv$ and express it in terms of z if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.

6) Find an analytic function $F(z) = u + iv$ if, $v = e^{-y}\sin x$ and $F(0) = 1$.

7) Find an analytic function $F(z) = u + iv$ where the real part is $e^{-2x}\sin (x^2 - y^2)$.

8) If $F(z) = u + iv$ is analytic function of $z = x + iy$ and $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$, find $F(z)$ if $f(\pi / 2) = 0$.

9) Show that the function $F(z) = e^{-y}\sin x$ is harmonic and find its harmonic conjugate.

10) Use Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ where $u = e^{x} (x\cos y - y\sin y)$.

11) Use Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ where $v = \tan^{-1} (y / x)$.

12) Determine the analytic function $F(z) = u + iv$ if $u = x^2 - y^2$ and $F(0) = 1$.

13) Find by Milne-Thomson's method the an analytic function $F(z) = u + iv$ where $v = e^{x} (x\sin y + y\cos y)$.

14) If $u = x^2 - y^2$ and $v = \dfrac{-y}{x^2 + y^2}$, then show that u and v satisfy Laplace equation but $u + iv$ is not an analytic function of z.

15) Show that if the harmonic functions u and v satisfy C.R. equations, then $u + iv$ is an analytic function.

16) If $F(z)$ is analytic in a simply connected region R then $\int_a^b F(z)\, dz$ is independent of the path of the integration in R joining the points a and b.

17) Evaluate $\int_C z\, dz$ where C is the arc of the parabola $y^2 = 4ax$ $(a > 0)$ in the first quadrant from the vertex to the end point of its latus rectum.

18) Evaluate $\int_C \dfrac{1}{z-a}\, dz$ where C is circle $|z - a| = 2$.

19) Evaluate $\int_C (y - x - 3x^2 i)\, dz$ where C is the straight line joining 0 to $1 + i$.

20) Evaluate $\int_C (y - x - 3x^2 i)\, dz$ where C is the straight line joining 0 to i first and then i to $1 + i$.

21) Show that the integral of $1 / z$ along a semicircular are from -1 to 1 has the value πi or $-\pi i$ according as the arc lies below or above the real axis.

22) Show that if F(z) is an analytic function in a region bounded by two simple closed curves C_1 and C_2 and also on C_1 and C_2, then $\int_{C_1} F(z)\,dz = \int_{C_2} F(z)\,dz$.

23) State Cauchy's theorem for integrals and verify it for $F(z) = z + 1$ rounder the contour $|z| = 1$.

24) If C is a circle $|z-a| = r$, prove that $\int_C (z-a)^n\,dz = 0$; n being an integer other than -1.

25) Evaluate $\int_C \frac{dz}{z}$ where C is the circle with centre at origin and radius a.

26) Verify Cauchy-Goursat Theorem for $F(z) = z + 2$ taken round the unit circle $|z| = 1$.

27) Verify Cauchy's integral Theorem for $F(z) = z^2$ round the circle $|z| = 1$.

28) Verify Cauchy's Theorem for $F(z) = z$ around a closed curve C. where c is the rectangle bounded by the lines : $x = 0$, $x = 1$, $y = 0$, $y= 1$,

29) Use Cauchy-Goursat Theorem to obtain the value of $\int_C e^2\,dz$, where C is the circle $|z| = 1$ and deduce that (i) $\int_0^{2\pi} e^{\cos\theta}\sin(\theta+\sin\theta)d\theta = 0$ (ii) $\int_0^{2\pi} e^{\cos\theta}\cos(\theta+\sin\theta)d\theta = 0$.

UNIT : 3 Cauchy's Integral Formulae and Residues.

I Questions of TWO marks

1) State Cauchy's integral formula for F(a).

2) State Cauchy's integral formula for $F'(a)$.

3) Evaluate by Cauchy's integral formula $\int_C \frac{z+2}{z}dz$ where C is the circle $|z| = 1$.

4) Evaluate $\int_{|z|=2} \frac{e^{2z}}{(z-1)^3}\,dz$.

5) Evaluate $\int_C \frac{ze^z}{(z-1)^3}\,dz$ where C is the circle $|z-1| = 2$.

6) Evaluate by Cauchy's integral formula $\int_C \frac{e^z}{z-2}dz$ where C is the circle $|z-2| = 2$.

7) Evaluate by Cauchy's integral formula $\int_C \frac{3z-1}{(z^2-2z-3)}dz$ where C is the circle $|z| = 4$.

8) Evaluate $\int_C \frac{z+3}{z^2=1}dz$ where C is the circle $|z| = 1/2$. Use Cauchy's integral formula.

9) Define apower series.

10) State Taylor's series for F(z) about $z = a$.

11) State Laurent's series for F(z) about $z = a$.

12) Expand in Taylor's series: $\frac{1}{z-2}$ for $|z| < 2$.

13) Expand in Laurent's series: F(z) $= \dfrac{1}{z-2}$ valid for $|z| < 2$.

14) Define zero of an analytic function.

15) Define singular point of an analytic function.

16) State the types of singularities.

17) Define a pole of an analytic function.

II Multiple Choice Questions 1 mark each

1) A power series R $= \sum_{n=0}^{\infty} a_n (z-a)^n$ converges if ………………

(a) $|z-a|$<R (b) $|z-a|$>R (c) $|z-a|$=R (d) None of these

2) If F(z) is an analytic function at z = a, then it has a power series expansion about z = a.
(a) Statement is true (b) Statement is false (d) None of these

3) The region of validity for Taylor's series about z = 0 of the function e^z is ………………
(a) $|z| = 0$ (b) $|z| < 1$ (c) $|z| < \infty$ (d) $|z| > 1$

4) The region of validity of $\dfrac{1}{1+z}$ for its Taylor's series expansion about z = 0 is ………………

(a) $|z| <1$ (b) $|z| > 1$ (c) $|z| = 1$ (d) None of these

5) The expansion of $\dfrac{1}{z-2}$ is valid for ………………

(a) $|z| <1$ (b) $|z| <2$ (c) $|z| > 3$ (d) None of these

6) If F(z) $= \dfrac{\sin z}{z}$, then z = 0 is its ………………………

(a) Removable singularity (b) Isolated singularity
(c) Essential singularity (d) None of these

7) Z = 1 is a ………….. of F(z) $= \dfrac{1}{z(z-1)^2}$.

(a) zero (b) simple pole (c) double pole (d) None of these

8) The residue of F(z) $= \dfrac{1+z}{z^2 - 2z^4}$ at a pole of order 2 is ………..

(a) 1 (b) - 1 (c) 2 (d) None of these

9) The singular points of F(z) $= \dfrac{1}{z(z-1)^2}$ are…………………..

(a) 0 , 1, -1 (b) 0 , 1, 1 (c) 1, -1 (d) None of these

III Questions of FOUR marks

1) State and prove Cauchy's integral formula for F(a).

2) State and prove Cauchy's integral formula for F′ (a).

3) Evaluate by Cauchy's integral formula $\int_C \dfrac{z+3}{z^2+1}$ dz , where C is

i) the circle $|z| = 2$

ii) the circle $|z| = ½$.

4) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z^2 - 3z + 2}$ where C is the circle $|z| = 3$

5) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^3 - 2z^2}$ dz, where C is the boundary of a square with vertices 1 + i, -1 + i , -1 - i and 1 – i traversed counter clock wise.

6) State Cauchy's integral formula for $F^n(a)$ and use it to evaluate $\int_{|z|=2} \frac{e^{2z}}{(z-1)^4}$ dz.

7) Evaluate $\int_{|z|=2} \frac{e^z}{z}$ dz. And hence deduce

i) $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta = 2\pi$ and ii) $\int_0^{2\pi} e^{\cos\theta} \sin(\sin\theta) d\theta = 0$

8) State Taylor's series for F(z) about z = a and find the Taylor's series expansion of F(z) = sinz in powers of z.

9) Evaluate $\int_{|z-1|=2} \frac{\sin \pi z}{(z-1)^2}$ dz.Expand in Taylors series

10) Expand in Taylor's series: $\frac{1}{z-2}$ for $|z| < 2$.

11) Expand in Taylor's series about z = 0 , the functions F(z) = $\frac{1}{1-z}$ and g(z) = coshz.

1 2) Expand in Taylor's series about z = 0 the following functions: (i) sin z ,(ii) sinh z , (iii) cos z.

13) Expand F(x) = e^z in Taylor's series expansion about z = 0. State the region of its validity.

14) Expand sinz in powers of (z - $\frac{\pi}{4}$).

15) Show that

$$\tan^{-1} z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + \ldots\ldots\ldots\ldots; |z| < 1.$$

16) Expand in Taylor's series:

F(z) = $\frac{1}{(z-1)(z-2)}$ for $|z| < 1$.

17) Expand F(z) = $\frac{1}{z-2}$ for $|z| < 2$ in Taylor's series.

18) Expand F(z) = $\frac{1}{z-2}$ in Laurent's series valid for $|z| < 2$.

19) Expand F(z) = $\frac{z^2-4}{(z^2+5z+4)}$ in powers of z for

(i) $|z| < 1$ (ii) $1 < |z| < 4$ and (iii) $|z| > 4$.

20) Expand $\frac{z^2-2z+5}{(z-2)(z^2+1)}$ on the annulus $1 < |z| < 2$.

21) Prove that $\frac{1}{4z-z^2}=\sum_{n=o}^{\infty}\frac{z^{n-1}}{4^{n+1}}$ where $0<|z|<4$.

1) Find poles and residues at these poles of f(z) = $\frac{1}{z.(z-1)^2}$ also find the sum of these residues.

2) Find the sum of residues at poles of f(z) = $\frac{e^z}{z^2+a^2}$

3) Find the residues of f(z) = $\frac{z^2}{(z-1)(z-2)(z-3)}$ at its poles.

4) Find the residues of $\frac{1}{(z^2+1)^3}$ at z = i.

5) Compute residues at double poles of f(z) = $\frac{z^2+2z+3}{(z-i)^2.(z+4)}$

6) Use Cauchy's integral formulae to evaluate (Any one)

i) $\int_C \frac{1}{(z^2+1)(z^2+4)}dz$, where C is the circle $|Z|=\frac{3}{2}$

ii) $\int_C \frac{dz}{z^3.(z+4)}$, Where C is the circle $|z|=2$.

iii) $\int_{|z-1|=2} \frac{ze^z}{(z-1)^3}dz$.

iv) $\int_C \frac{dz}{(z^2+4)^2}$, Where C is the circle $|z-i|=2$.

7) Show that $\int_C \frac{e^{2z}}{(z+1)^4}dz=\frac{8\Pi ie^{-2}}{3}$, where C is the circle $|z|=3$.

8) Expand f(z) = $\frac{z^2-1}{(z+2)(z+3)}$ in the regions:

i) $|z|\langle 2$, ii) $2\langle|z|\langle 3$, iii) $|z|\rangle 3$

30) Expand : $\frac{1}{z^2-3z+2}$ for

i) $0\langle|z|\langle 1$, ii) $1\langle|z|\langle 2$ and iii) $|z|\rangle 2$

Unit -4

Cauchy's Residue Theorem and Contour Integration

I) **Questions of Two Marks ;**

1) State Cauchy's Residue Theorem.

2) find all poles of f(z) = $\dfrac{3z^2+2}{(z-1)(z^2+9)}$

3) Find the residues of f(z) a z =0,

Where, f(z) = $\dfrac{e^z}{z(z-1)^2}$.

4) Define a rational function.

5) Find the residues of f(z) = . $\dfrac{3z^2+2}{(z-1)(z^2+9)}$

6) Find Zeros and poles of f(z) = $\dfrac{e^z}{z(z-1)^2}$

7) Find all zeros and poles of f(z) = $\dfrac{z^2-2z}{(z+1)^2(z^2+4)}$

8) Classify the poles of f(z) = $\dfrac{1}{z^3(z+4)}$

9) Which of the poles of f(z) = $\dfrac{1}{(3z+1)(z+3)}$

Lies inside the circle $|z|=1$.

7) Which of the poles of f(z) = $\dfrac{1}{z^2+1}$ lies in the upper half of the z- plane.

8) Find the poles of f(z) = $\dfrac{1}{(z^2+a^2)(z^2+b^2)}$ which lie in the lower half of the complex plane.

9) Find all zeros and poles of f(z) = $\dfrac{z^2}{(z^2+1)(z^2+4)}$ and Classify them.

10) Find all zeros and poles of $\dfrac{\cos x}{x^2+1}$

11) Find all zeros and poles of $\dfrac{x^3.\sin x}{(x^2+a^2)(x^2+b^2)}$

III) Questions of Six Marks :

1) State and prove Cauchy's Residue Theorem.

2) Evaluate by Cauchy Residue Theorem : $\int_C \dfrac{5z-2}{z.(z-1)}dz$, where Cis the

Circle $|z| = 2$ taken Counter clockwise.

3)Evaluate: $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ by Cauchy's Residue Theorem , where C is

i) the circle $|z-2| = 2$,

ii) the circle $|z| = 4$

4)Evaluate : $\int_C \frac{e^z}{z(z-1)^2} dz$, where C is the circle $|z| = 3$ traversed in positive direction,

5) Evaluate : $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ by Cauchy's Residue Theorem , where Cis the

rectangle formed by the lines x=$\pm 2, y = \pm 3$.

6) Use Cauchy's residue theorem to evaluate $\int_{|z|=2} \frac{dz}{z^3.(z+4)}$

7) Use Contour integration to evaluate $\int_0^{2\Pi} \frac{d\theta}{5+3\cos\theta}$

8) Evaluate : $\int_0^{2\Pi} \frac{d\theta}{5+3\sin\theta}$

9) Evaluate : $\int_0^{2\Pi} \frac{d\theta}{(\cos\theta+2)^2}$

10) Use method of contour integration to evaluate $\int_0^{\Pi} \frac{2d\theta}{4+\sin^2\theta}$

11) Apply calculus of residues to evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$

12) Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$

13) Evaluate : $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$; where a$\rangle 0, b \rangle 0$

14) By Contour integration , evaluate $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$

15) Evaluate : $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+1} dx$ by using Contour integration.

16) Evaluate by contour integration , $\int_{0}^{\infty} \frac{x^3 . \sin x}{(x^2+a^2)(x^2+b^2)} dx$ where $a \rangle 0, b \rangle 0$.

17) Evaluate by Cauchy's residues theorem $\int_{|z|=1} \frac{e^{-z}}{z^2} dz$

18) Evaluate by contour integration $\int_{0}^{2\Pi} \frac{d\theta}{5+4\sin\theta}$

19) Evaluate by Contour integration $\int_{0}^{\Pi} \frac{d\theta}{3+2\cos\theta}$

20) Evaluate $\int_{0}^{2\Pi} \frac{d\theta}{3+2\cos\theta+\sin\theta}$

21) Evaluate : $\int_{-\Pi}^{\Pi} \frac{\cos}{5+4\cos\theta} d\theta$

22) Evaluate : $\int_{-\infty}^{\infty} \frac{dx}{x^4+13x^2+36}$

23) Evaluate , $\int_{-\infty}^{\infty} \frac{dx}{x^2+x+1}$

24) Evaluate : $\int_{0}^{\infty} \frac{dx}{(x^2+1)^2}$

25) Evaluate ; $\int_{0}^{\infty} \frac{dx}{(x^4-6x^2+25)}$

26) Evaluate : $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+4} dx$

27) Evaluate : $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2+a^2} dx$

28) Use Contour integration to prove that $\int_{-\Pi}^{\Pi} \frac{d\theta}{1+\sin^2\theta} = \Pi\sqrt{2}$

29) Show that $\int_{0}^{2\Pi} \frac{d\theta}{a+b\cos\theta} = \int_{0}^{2\Pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\Pi}{\sqrt{a^2-b^2}}$ where $a \rangle 0, b \rangle 0$

30) Prove that $\int_{0}^{\infty} \frac{\cos mx}{x^2+a^2} dx = \frac{\Pi}{2a} e^{-ma}, m \geq 0$ and $a \rangle 0$

31) Prove that, $\int_{-\infty}^{\infty} \frac{x.\sin ax}{x^2+4} dx = \frac{\Pi}{2} e^{-a} \sin a; a \rangle 0$

I) Multipal Choice Questions;

1) The poles of f(z) = $\frac{e^z}{z^2+a^2}$ are .- - - - -

a) $\pm 2i$,b) 0,1 , c) $\pm ai$, d) None of these.

2) The poles of f(z) = $\frac{1}{\left(z^2+1\right)^3}$ are

a) $\pm 3i$, b) 2,3 ,c) $\pm i$,d) None of these.

3) The sum of the residues at poles of f(z) = $\frac{e^z}{z^2+a^2}$ is

a) $\frac{1}{a}\sin a$, b) $-\frac{1}{2}$, c) $\frac{3}{2}$, d) None of these.

4) The sum of the residues of f(z) = $\frac{1}{\left(z^2+1\right)^3}$ is ------

a) 0, b)1, c) -1,d) None of these.

5) The residue of f(z) = $\frac{1+z}{z^2-2z^4}$ at z = 0 is

a) 1 , b) 0 , c) -1 , d) None of these.

6) The sum of residues at its poles of f(z) = $\frac{1}{z(z-1)^2}$ is -- - - - -

a) 1 , b) 0 , c) -1 , d) None of these.

7) The simple poles of f(z) = $\frac{z^2-4}{z^2+5z+4}$ are

a) 1,4 b)-1,4 c) -1,-4 d) None of these.

8) For the function f(z) = $\frac{z^2+3}{z^2.\left(z^2+4\right)}$, the pole z=0 has order - - - - - -

a) 1, b)2, c)0 , d) None of these.