

North Maharashtra University ; Jalgaon.

Question Bank

S.Y.B.Sc. Mathematics (Sem –II)

MTH – 221 . Functions of a Complex Variable.

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Unit – 1 :

Functions of a Complex Variable.

I) Questions of Two marks :

1) The $\lim_{z \rightarrow 2+3i} [3x+i(2x-4y)]$ is -----

2) Does $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$ exist

3) What are the points of discontinuities of $f(z) = \frac{2z-3}{z^2-2z+2}$

4) Write the real and imaginary parts of $f(z) = z^3$ where $z = x+iy$.

5) Find the limit, $\lim_{z \rightarrow 1-i} [x+i(2x+y)]$

1) Does Continuity at a point imply differentiability there at. Justify by an example.

2) Define an analytic function .

3) Define singular points of an analytic function $f(z)$.

4) Find the singular points for the function $f(z) = \frac{z-2}{(z+1)(z^2+1)}$

5) Define a Laplace's Didifferential Equation for $\Phi(x, y)$.

6) What is harmonic function ?

7) What do you mean by $f(z)$ is differentiable at ?

8) Is the function $u = \frac{1}{2} \cdot \log(x^2 + y^2)$ harmonic?

9) When do you say $f(z)$ tends to a limit as z tends to z_0 ?

10) State Cauchy- Riemann equations.

11) State the necessary condition for the function $f(x)$ to be abalytic.

12) Every differential function is continuous . True or False.

II) Multiple Choice Questions :

1) If $\lim_{z \rightarrow 1-i} [x+i(2x+y)] = p+iq$, then $(p,q) = \text{-----}$.

- (i) (1,1) (ii) (-1,1) (iii) (1,-1) (iv) (-1,-1)

2) The function $f(z) = \frac{xy}{x^2 + y^2}$ when $z \neq 0$ and $f(0) = 0$ is

(i) Continuous at $z = 0$, (ii) Discontinuous at $z = 0$

(iii) Not predictable, (iv) Constant

3) A Continous function is iffereential :

(i) True, (ii) False.

(iii) True & False, (v) True or False

1) A function $\Phi(x, y)$ satisfying Laplace equation is called

(i) Analytic (ii) Holomorphic

(iii) Harmonic, (iv) Non-harmonic

2) A function $f(z) = e^z$ is

(i) Analytic everywhere, (ii) Analytic nowhere

(iii) only differentiable, (iv) None

3) If $f(z) = u - iv$ is analytic in the z -plane, then the C-R equations satisfied by its real and imaginary parts are,

(i) $u_x = u_y$; $u_y = -v_x$ (ii) $u_x = -v_y, u_y = v_x$

7) An analytic function with constant modulus is

(a) Constant, (b) not constant, (c) analytic, (d) None of these.

8) A Milne – Thomson method is used to construct

a) analytic function, b) Continuous function

c) differentiable function, d) None of these.

III) Questions for Four marks ;

1) Define the continuity of $f(z)$ at $z = z_0$ and examine for continuity at $z=0$ the function

$$f(z) = \frac{x^4 y(iy - x)}{(x^8 + y^2)z}; z \neq 0$$

1) Define limit of a function $f(z)$.

Evaluate ; $\lim_{z \rightarrow \frac{2\pi i}{3}} \frac{z^3 + 8}{z^4 + 4z^2 + 16}$

2) Prove that a differentiable function is always continuous. Is the converse true? Justify by an example.

3) Use the definition of limit to prove that, $\lim_{z \rightarrow 1-i} [x + i(2x + y)] = 1 + i$

5) Show that if $\lim f(z)$ exists, it is unique

$$z \rightarrow z_0$$

6) Prove that $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$ does not exist

4) Prove that $\lim_{z \rightarrow 0} \frac{x^3 y(y - ix)}{(x^6 + y^2).z}$ does not exist, where $z \neq 0$

5) Evaluate : $\lim_{z \rightarrow i} \frac{z^5 - i}{z + i}$

6) 9) Evaluate : $\lim_{z \rightarrow 1+i} \frac{z^4 + 4}{z - 1 - i}$

10) Examine for continuity the function, $f(z) = \frac{z^4 + 4}{z - 2i}$ at $z = 2i$

$$= 3+4i ; z=2i, \text{ at } z = 2i$$

11) If $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ when $z \neq i$ and $f(i) = 2+3i$, examine $f(z)$ for continuity

At $z=i$.

12) Show that the function $f(z) = \bar{z}$ is continuous everywhere but not differentiable.

13) Define an analytic function. Give two examples of an analytic function.

14) Show that $f(z) = |z|^2$ is not analytic at any point in the z -plane.

- 15) State and prove the necessary condition for the $f(z)$ to be analytic . Are these conditions sufficient ?
- 16) State and prove the sufficient conditions for the function $f(z)$ to be analytic.
- 17) Prove that a necessary condition for a complex function $w = f(z) = u(x,y)+iv(x,y)$ to be analytic at a point $z =x+iy$ of its domain D is that at (x,y) the first order partial derivatives of u and v with respect to x and y exist and satisfy the Cauchy – Riemann equations : $u_x = v_y$ and $u_y = -v_x$.
- 18) Prove that for the function $F(z) = U(x,y) + V(x, y)$, if the four partial derivatives U_x, U_y, V_x and V_y exist and are continuous at a point $z = x + iy$ in the domain D and that they satisfy Cauchy-Riemann equations: $U_x = V_y$; $U_y = -V_x$ at (x, y) , then $F(z)$ is analytic at the point $z = x + iy$.
- 19) Show that the function defined by $F(z) = \sqrt{|xy|}$, when $z \neq 0$ and $F(0) = 0$, is not analytic at $z = 0$ even though the C-R equations are satisfied at $z = 0$.
- 20) Define $F(z) = z^5 |z|^4$; if $z \neq 0$
 $= 0$; if $z = 0$.
 Show that $F(z)$ is not analytic at the origin even though it satisfies C-R equations at the origin.
- 21) Show that the function $F(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ when $z \neq 0$ and $F(0) = 0$ is continuous at $z = 0$ and C-R equations are satisfied at the origin.
- 22) If $F(z)$ and $\overline{F(\overline{z})}$ are analytic functions of z , show that $F(z)$ is a constant function.
- 23) If $F(z)$ is an analytic function with constant modulus, then prove that $F(z)$ is a constant function.
- 24) Show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \overline{z}}$.
- 25) Show that $F(z) = e^{\overline{z}}$ is not analytic for any z .
- 26) Show that $\lim_{z \rightarrow 0} \frac{x^2 y}{x^4 + y^2}$ does not exist.
- 27) Show that if $W = F(z) = 3x - 2iy$, then $\frac{dW}{dz}$ does not exist.
- 28) Show that the function $F(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ when $z \neq 0$ and $F(0) = 0$ is continuous and that C-R equations are satisfied at the origin but $F'(0)$ does not exist .
 Show that the function defined by $F(z) = \frac{xy^2(x+iy)}{x^2 + y^4}$; $z \neq 0$
 $= 0$; $z = 0$.
 satisfies C.R. equations at $z = 0$ but not analytic there at.
 If $F(z)$ is an analytic function of z then show that
- 30) (i) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} |F(z)|^2 = 4 |F'(z)|^2$
 (ii) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} [RF(z)]^2 = 2 |F'(z)|^2$

I Questions of TWO marks

- 1) Define Laplace Differential equation.
- 2) Define harmonic and conjugate harmonic functions.
- 3) True or False:
 - i) If $F(z)$ is an analytic function of z , then $F(z)$ depends on \bar{z} .
 - ii) If $F(z)$ and $\overline{F(\bar{z})}$ are analytic functions of z , then $F(z)$ is a constant.
 - iii) An analytic function with constant modulus is constant.
- 4) Is $u = x^2 - y^2$ a harmonic function? Justify.
- 5) Show that $v(x, y) = x^2 - y^2 + x$ is harmonic function.
- 6) Show that $u(x, y) = e^{-y}\sin x$ is a harmonic function.
- 7) Prove or disprove: $u = y^3 - 3x^2y$ is a harmonic function.
- 8) Show that $v = x^3 - 3xy^2$ satisfies Laplace's differential equation.
- 9) State Cauchy-Goursat Theorem.
- 10) Define simple closed curve.
- 11) Define the term Simply connected region.
- 12) Define Jordan Curve.
- 13) State Jordan Curve theorem.
- 14) Evaluate $\int_C \frac{1}{z-a} dz$ where C is circle $|z-a|=2$.
- 15) Evaluate $\int_0^{3+i} z^2 dz$ along the line $x = 3y$.

II Multiple Choice questions 1 mark each

- 1) The harmonic conjugate of $e^x \cos y$ is
 (a) $e^x \cos y + c$ (b) $e^x \sin y + c$ (c) $e^x + c$ (d) None of these
- 2) The harmonic conjugate of $e^{-y} \sin x$ is
 (a) $e^{-y} \cos x + c$ (b) $e^{-y} \sin x + c$ (c) $e^{-x} \cos y$ (d) None of these
- 3) The value of the integral $\int_C (12z^2 - 4/z) dz$ where C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining points (1,1) and (2,3) is given by
 (a) $-156 + 58i$ (b) $-156 - 58i$ (c) 50 (d) None of these
- 4) The value of $\int_0^1 z e^{2z} dz$ will be
 (a) e (b) $1/4 (e^2 + 1)$ (c) $1/4 (e^2 - 1)$ (d) None of these
- 5) The value of the integral of $1/z$ along a semicircular arc from -1 to 1 in the clockwise direction will be
 (a) zero (b) $-\pi i$ (c) πi (d) None of these

Questions of THREE marks

- 1) If $F(z) = u + iv$ is an analytic function then show that u and v both satisfy Laplace's differential equation.
- 2) If $F(z) = u(x,y) + iv(x,y)$ is an analytic function, show that $F(z)$ is independent of \bar{z} .
- 3) Explain the Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ when the real part u is given.
- 4) Explain the Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ when the imaginary part v is given.
- 5) Find an analytic function $F(z) = u + iv$ and express it in terms of z if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.
- 6) Find an analytic function $F(z) = u + iv$ if, $v = e^{-y}\sin x$ and $F(0) = 1$.
- 7) Find an analytic function $F(z) = u + iv$ where the real part is $e^{-2x}\sin(x^2 - y^2)$.
- 8) If $F(z) = u + iv$ is analytic function of $z = x + iy$ and $u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$, find $F(z)$ if $f(\pi/2) = 0$.
- 9) Show that the function $F(z) = e^{-y}\sin x$ is harmonic and find its harmonic conjugate.
- 10) Use Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ where $u = e^x (x \cos y - y \sin y)$.
- 11) Use Milne-Thomson's method to construct an analytic function $F(z) = u + iv$ where $v = \tan^{-1}(y/x)$.
- 12) Determine the analytic function $F(z) = u + iv$ if $u = x^2 - y^2$ and $F(0) = 1$.
- 13) Find by Milne-Thomson's method the an analytic function $F(z) = u + iv$ where $v = e^x (x \sin y + y \cos y)$.
- 14) If $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$, then show that u and v satisfy Laplace equation but $u + iv$ is not an analytic function of z .
- 15) Show that if the harmonic functions u and v satisfy C.R. equations, then $u + iv$ is an analytic function.
- 16) If $F(z)$ is analytic in a simply connected region R then $\int_a^b F(z) dz$ is independent of the path of the integration in R joining the points a and b .
- 17) Evaluate $\int_C z dz$ where C is the arc of the parabola $y^2 = 4ax$ ($a > 0$) in the first quadrant from the vertex to the end point of its latus rectum.
- 18) Evaluate $\int_C \frac{1}{z-a} dz$ where C is circle $|z-a|=2$.
- 19) Evaluate $\int_C (y - x - 3x^2 i) dz$ where C is the straight line joining 0 to $1 + i$.
- 20) Evaluate $\int_C (y - x - 3x^2 i) dz$ where C is the straight line joining 0 to i first and then i to $1 + i$.
- 21) Show that the integral of $1/z$ along a semicircular arc from -1 to 1 has the value πi or $-\pi i$ according as the arc lies below or above the real axis.

- Show that if $F(z)$ is an analytic function in a region bounded by two simple closed curves C_1 and C_2 and also on C_1 and C_2 , then $\int_{C_1} F(z) dz = \int_{C_2} F(z) dz$.
- 22) State Cauchy's theorem for integrals and verify it for $F(z) = z + 1$ rounder the contour $|z| = 1$.
- 23) If C is a circle $|z - a| = r$, prove that $\int_C (z - a)^n dz = 0$; n being an integer other than -1 .
- 24) Evaluate $\int_C \frac{dz}{z}$ where C is the circle with centre at origin and radius a .
- 25) Verify Cauchy-Goursat Theorem for $F(z) = z + 2$ taken round the unit circle $|z| = 1$.
- 26) Verify Cauchy's integral Theorem for $F(z) = z^2$ round the circle $|z| = 1$.
- 27) Verify Cauchy's Theorem for $F(z) = z$ around a closed curve C . where c is the rectangle bounded by the lines : $x = 0, x = 1, y = 0, y = 1$,
- Use Cauchy-Goursat Theorem to obtain the value of $\int_C e^z dz$, where C is the circle $|z| = 1$ and
- 28) deduce that (i) $\int_0^{2\pi} e^{\cos\theta} \sin(\theta + \sin\theta) d\theta = 0$ (ii) $\int_0^{2\pi} e^{\cos\theta} \cos(\theta + \sin\theta) d\theta = 0$.

UNIT : 3 Cauchy's Integral Formulae and Residues.
I Questions of TWO marks

- 1) State Cauchy's integral formula for $F(a)$.
- 2) State Cauchy's integral formula for $F'(a)$.
- 3) Evaluate by Cauchy's integral formula $\int_C \frac{z+2}{z} dz$ where C is the circle $|z| = 1$.
- 4) Evaluate $\int_{|z|=2} \frac{e^{2z}}{(z-1)^3} dz$.
- 5) Evaluate $\int_C \frac{ze^z}{(z-1)^3} dz$ where C is the circle $|z-1| = 2$.
- 6) Evaluate by Cauchy's integral formula $\int_C \frac{e^z}{z-2} dz$ where C is the circle $|z-2| = 2$.
- 7) Evaluate by Cauchy's integral formula $\int_C \frac{3z-1}{(z^2-2z-3)} dz$ where C is the circle $|z| = 4$.
- 8) Evaluate $\int_C \frac{z+3}{z^2-1} dz$ where C is the circle $|z| = 1/2$. Use Cauchy's integral formula.
- 9) Define a power series.
- 10) State Taylor's series for $F(z)$ about $z = a$.
- 11) State Laurent's series for $F(z)$ about $z = a$.
- 12) Expand in Taylor's series: $\frac{1}{z-2}$ for $|z| < 2$.

- 13) Expand in Laurent's series: $F(z) = \frac{1}{z-2}$ valid for $|z| < 2$.
- 14) Define zero of an analytic function.
- 15) Define singular point of an analytic function.
- 16) State the types of singularities.
- 17) Define a pole of an analytic function.

II Multiple Choice Questions 1 mark each

- 1) A power series $R = \sum_{n=0}^{\infty} a_n(z-a)^n$ converges if
 - (a) $|z-a| < R$ (b) $|z-a| > R$ (c) $|z-a| = R$ (d) None of these
- 2) If $F(z)$ is an analytic function at $z = a$, then it has a power series expansion about $z = a$.
 - (a) Statement is true (b) Statement is false (d) None of these
- 3) The region of validity for Taylor's series about $z = 0$ of the function e^z is
 - (a) $|z| = 0$ (b) $|z| < 1$ (c) $|z| < \infty$ (d) $|z| > 1$
- 4) The region of validity of $\frac{1}{1+z}$ for its Taylor's series expansion about $z = 0$ is
 - (a) $|z| < 1$ (b) $|z| > 1$ (c) $|z| = 1$ (d) None of these
- 5) The expansion of $\frac{1}{z-2}$ is valid for
 - (a) $|z| < 1$ (b) $|z| < 2$ (c) $|z| > 3$ (d) None of these
- 6) If $F(z) = \frac{\sin z}{z}$, then $z = 0$ is its
 - (a) Removable singularity (b) Isolated singularity
 - (c) Essential singularity (d) None of these
- 7) $Z = 1$ is a of $F(z) = \frac{1}{z(z-1)^2}$.
 - (a) zero (b) simple pole (c) double pole (d) None of these
- 8) The residue of $F(z) = \frac{1+z}{z^2-2z^4}$ at a pole of order 2 is
 - (a) 1 (b) -1 (c) 2 (d) None of these
- 9) The singular points of $F(z) = \frac{1}{z(z-1)^2}$ are.....
 - (a) 0, 1, -1 (b) 0, 1, 1 (c) 1, -1 (d) None of these

III Questions of FOUR marks

- 1) State and prove Cauchy's integral formula for $F(a)$.
- 2) State and prove Cauchy's integral formula for $F'(a)$.

Evaluate by Cauchy's integral formula $\int_C \frac{z+3}{z^2+1} dz$, where C is

- 3)
 - i) the circle $|z| = 2$
 - ii) the circle $|z| = \frac{1}{2}$.

- 4) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z^2 - 3z + 2} dz$ where C is the circle $|z| = 3$
- 5) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^3 - 2z^2} dz$, where C is the boundary of a square with vertices $1 + i$, $-1 + i$, $-1 - i$ and $1 - i$ traversed counter clock wise.
- 6) State Cauchy's integral formula for $F^n(a)$ and use it to evaluate $\int_{|z|=2} \frac{e^{2z}}{(z-1)^4} dz$.
- 7) Evaluate $\int_{|z|=2} \frac{e^z}{z} dz$. And hence deduce
- i) $\int_0^{2\pi} e^{\cos \theta} \cos(\sin \theta) d\theta = 2\pi$ and ii) $\int_0^{2\pi} e^{\cos \theta} \sin(\sin \theta) d\theta = 0$
- 8) State Taylor's series for $F(z)$ about $z = a$ and find the Taylor's series expansion of $F(z) = \sin z$ in powers of z .
- 9) Evaluate $\int_{|z-1|=2} \frac{\sin \pi z}{(z-1)^2} dz$. Expand in Taylor's series
- 10) Expand in Taylor's series: $\frac{1}{z-2}$ for $|z| < 2$.
- 11) Expand in Taylor's series about $z = 0$, the functions $F(z) = \frac{1}{1-z}$ and $g(z) = \cosh z$.
- 12) Expand in Taylor's series about $z = 0$ the following functions: (i) $\sin z$, (ii) $\sinh z$, (iii) $\cos z$.
- 13) Expand $F(x) = e^z$ in Taylor's series expansion about $z = 0$. State the region of its validity.
- 14) Expand $\sin z$ in powers of $(z - \frac{\pi}{4})$.
- 15) Show that $\tan^{-1} z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + \dots; |z| < 1$.
- 16) Expand in Taylor's series: $F(z) = \frac{1}{(z-1)(z-2)}$ for $|z| < 1$.
- 17) Expand $F(z) = \frac{1}{z-2}$ for $|z| < 2$ in Taylor's series.
- 18) Expand $F(z) = \frac{1}{z-2}$ in Laurent's series valid for $|z| < 2$.
- 19) Expand $F(z) = \frac{z^2 - 4}{(z^2 + 5z + 4)}$ in powers of z for
- (i) $|z| < 1$ (ii) $1 < |z| < 4$ and (iii) $|z| > 4$.
- 20) Expand $\frac{z^2 - 2z + 5}{(z-2)(z^2 + 1)}$ on the annulus $1 < |z| < 2$.

21) Prove that $\frac{1}{4z - z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$ where $0 < |z| < 4$.

1) Find poles and residues at these poles of $f(z) = \frac{1}{z \cdot (z-1)^2}$ also find the sum of these residues.

2) Find the sum of residues at poles of $f(z) = \frac{e^z}{z^2 + a^2}$

3) Find the residues of $f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$ at its poles.

4) Find the residues of $\frac{1}{(z^2 + 1)^3}$ at $z = i$.

5) Compute residues at double poles of $f(z) = \frac{z^2 + 2z + 3}{(z-i)^2 \cdot (z+4)}$

6) Use Cauchy's integral formulae to evaluate (Any one)

i) $\int_C \frac{1}{(z^2 + 1)(z^2 + 4)} dz$, where C is the circle $|z| = \frac{3}{2}$

ii) $\int_C \frac{dz}{z^3 \cdot (z+4)}$, Where C is the circle $|z| = 2$.

iii) $\int_{|z-1|=2} \frac{ze^z}{(z-1)^3} dz$.

iv) $\int_C \frac{dz}{(z^2 + 4)^2}$, Where C is the circle $|z-i| = 2$.

7) Show that $\int_C \frac{e^{2z}}{(z+1)^4} dz = \frac{8\pi i e^{-2}}{3}$, where C is the circle $|z| = 3$.

8) Expand $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in the regions:

i) $|z| < 2$, ii) $2 < |z| < 3$, iii) $|z| > 3$

30) Expand: $\frac{1}{z^2 - 3z + 2}$ for

i) $0 < |z| < 1$, ii) $1 < |z| < 2$ and iii) $|z| > 2$

Unit -4

Cauchy's Residue Theorem and Contour Integration

I) Questions of Two Marks ;

1) State Cauchy's Residue Theorem.

2) find all poles of $f(z) = \frac{3z^2 + 2}{(z-1)(z^2 + 9)}$

3) Find the residues of $f(z)$ at $z=0$,

Where, $f(z) = \frac{e^z}{z(z-1)^2}$.

4) Define a rational function.

5) Find the residues of $f(z) = \frac{3z^2 + 2}{(z-1)(z^2 + 9)}$

6) Find Zeros and poles of $f(z) = \frac{e^z}{z(z-1)^2}$

7) Find all zeros and poles of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2 + 4)}$

8) Classify the poles of $f(z) = \frac{1}{z^3(z+4)}$

9) Which of the poles of $f(z) = \frac{1}{(3z+1)(z+3)}$

Lies inside the circle $|z|=1$.

7) Which of the poles of $f(z) = \frac{1}{z^2 + 1}$ lies in the upper half of the z - plane.

8) Find the poles of $f(z) = \frac{1}{(z^2 + a^2)(z^2 + b^2)}$ which lie in the lower half of the complex plane.

9) Find all zeros and poles of $f(z) = \frac{z^2}{(z^2 + 1)(z^2 + 4)}$ and Classify them.

10) Find all zeros and poles of $\frac{\cos x}{x^2 + 1}$

11) Find all zeros and poles of $\frac{x^3 \cdot \sin x}{(x^2 + a^2)(x^2 + b^2)}$

III) Questions of Six Marks :

1) State and prove Cauchy's Residue Theorem.

2) Evaluate by Cauchy Residue Theorem : $\int_C \frac{5z-2}{z(z-1)} dz$, where C is the

Circle $|z|=2$ taken Counter clockwise.

3) Evaluate: $\int_C \frac{3z^2 + 2}{(z-1)(z^2 + 9)} dz$ by Cauchy's Residue Theorem, where C is

i) the circle $|z-2|=2$,

ii) the circle $|z|=4$

4) Evaluate: $\int_C \frac{e^z}{z(z-1)^2} dz$, where C is the circle $|z|=3$ traversed in positive direction,

5) Evaluate: $\int_C \frac{z^2 - 2z}{(z+1)^2(z^2 + 4)} dz$ by Cauchy's Residue Theorem, where C is the

rectangle formed by the lines $x=+2, y=+3$.

6) Use Cauchy's residue theorem to evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$

7) Use Contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$

8) Evaluate: $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$

9) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(\cos\theta + 2)^2}$

10) Use method of contour integration to evaluate $\int_0^{\pi} \frac{2d\theta}{4 + \sin^2\theta}$

11) Apply calculus of residues to evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$

12) Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

13) Evaluate: $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$; where $a>0, b>0$

14) By Contour integration, evaluate $\int_0^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$

15) Evaluate: $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 1} dx$ by using Contour integration.

16) Evaluate by contour integration , $\int_0^{\infty} \frac{x^3 \cdot \sin x}{(x^2 + a^2)(x^2 + b^2)} dx$ where $a > 0, b > 0$.

17) Evaluate by Cauchy's residues theorem $\int_{|z|=1} \frac{e^{-z}}{z^2} dz$

18) Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$

19) Evaluate by Contour integration $\int_0^{\pi} \frac{d\theta}{3 + 2 \cos \theta}$

20) Evaluate $\int_0^{2\pi} \frac{d\theta}{3 + 2 \cos \theta + \sin \theta}$

21) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos \theta}{5 + 4 \cos \theta} d\theta$

22) Evaluate : $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 13x^2 + 36}$

23) Evaluate , $\int_{-\infty}^{\infty} \frac{dx}{x^2 + x + 1}$

24) Evaluate : $\int_0^{\infty} \frac{dx}{(x^2 + 1)^2}$

25) Evaluate ; $\int_0^{\infty} \frac{dx}{(x^4 - 6x^2 + 25)}$

26) Evaluate : $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 4} dx$

27) Evaluate : $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2} dx$

28) Use Contour integration to prove that $\int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2 \theta} = \pi\sqrt{2}$

29) Show that $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \int_0^{2\pi} \frac{d\theta}{a + b \sin \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ where $a > 0, b > 0$

30) Prove that $\int_0^{\infty} \frac{\cos mx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-ma}$, $m \geq 0$ and $a > 0$

31) Prove that, $\int_{-\infty}^{\infty} \frac{x \cdot \sin ax}{x^2 + 4} dx = \frac{\pi}{2} e^{-a} \sin a; a > 0$

D) Multiple Choice Questions;

1) The poles of $f(z) = \frac{e^z}{z^2 + a^2}$ are .-----

a) $\pm 2i$, b) $0,1$, c) $\pm ai$, d) None of these.

2) The poles of $f(z) = \frac{1}{(z^2 + 1)^3}$ are

a) $\pm 3i$, b) $2,3$, c) $\pm i$, d) None of these.

3) The sum of the residues at poles of $f(z) = \frac{e^z}{z^2 + a^2}$ is

a) $\frac{1}{a} \sin a$, b) $-\frac{1}{2}$, c) $\frac{3}{2}$, d) None of these.

4) The sum of the residues of $f(z) = \frac{1}{(z^2 + 1)^3}$ is -----

a) 0 , b) 1, c) -1, d) None of these.

5) The residue of $f(z) = \frac{1+z}{z^2 - 2z^4}$ at $z = 0$ is

a) 1 , b) 0 , c) -1 , d) None of these.

6) The sum of residues at its poles of $f(z) = \frac{1}{z(z-1)^2}$ is -----

a) 1 , b) 0 , c) -1 , d) None of these.

7) The simple poles of $f(z) = \frac{z^2 - 4}{z^2 + 5z + 4}$ are

a) 1,4 b)-1,4 c) -1,-4 d) None of these.

8) For the function $f(z) = \frac{z^2 + 3}{z^2 \cdot (z^2 + 4)}$, the pole $z=0$ has order -----

a) 1, b) 2, c) 0 , d) None of these.