

North Maharashtra University , Jalgaon

Question Bank

(New Syllabus w.e.f. June 2008)

Class- S. Y.B.Sc.

Subject : Mathematics

Paper MTH-211

(Calculus of Several Variables)

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Unit -I (Functions of Two & Three Variables)

I) Objective Type Questions (2 Marks each)

- 1) Define neighbourhood of a point in a plane .
- 2) Define simultaneous limit of a function $f(x,y)$ as $(x,y) \rightarrow (a,b)$
- 3) Define Continuity of a function $f(x,y)$ at a point (a,b)
- 4) Find two repeated limits of $f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$, $(x,y) \neq (0,0)$.

As $x \rightarrow 0, y \rightarrow 0$

- 5) Evaluate the limit,

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6} \quad \text{if exists}$$

- 6) Define $f_x(a,b)$ and $f_y(a,b)$
- 7) Define $f_{xx}(a,b)$ and $f_{yy}(a,b)$
- 8) Define $f_{xy}(a,b)$ and $f_{yx}(a,b)$
- 9) If $u = \tan^{-1} \frac{y}{x}$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$
- 10) Find $f_x(x,y)$ and $f_y(x,y)$ if $f(x,y) = e^x \sin xy$
- 11) If $u(x,y) = \frac{x}{y^2} - \frac{y}{x^2}$ find $u_x(1,1)$ and $u_y(1,1)$.
- 12) Define differentiability of a function $f(x,y)$ at a point (a,b) of its domain.
- 13) State the necessary condition for differentiability of a function $f(x,y)$ at a point (a,b) .
- 14) State sufficient condition for differentiability of a function $f(x,y)$ at a point (a,b)
- 15) State Young's theorem for the equality of f_{xy} and f_{yx}
- 16) State Schwarz's theorem for the equality of f_{xy} and f_{yx}

II) Multiple Choice Questions (1 Marks each)

Choose the correct option .

- 1) If a function $f(x,y)$ is discontinuous at a point (a,b) then
 - a) $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$ exist and equal to $f(a,b)$
 - b) $f(x,y)$ is not differentiable at (a,b)
 - c) $f(x,y)$ is differentiable at (a,b)
 - d) None of these.
- 2) If the simultaneous limit exists and has the same value along any three different paths then
 - a) $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$ exists.
 - b) $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$ may or may not exist
 - c) $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$ does not exist

- d) None of these.
- 3) If a function $f(x,y)$ is differentiable at a point (a,b) then,
- $f_x(a,b)$ & $f_y(a,b)$ may or maynot exist.
 - $f_x(a,b)$ & $f_y(a,b)$ both exist .
 - only one of $f_x(a,b)$ and $f_y(a,b)$ exists.
 - $f_x(a,b)$ and $f_y(a,b)$ both does not exist.
- 4) If $f(x,y) = x^3 + y^3 - 2x^2y^2$ then
 $f(x,x) (1,1) = \text{-----}$
- 1
 - 1
 - 0
 - None of these.

III) Theory and Examples (4- Marks each)

- If a function $f(x,y)$ is differentiable at a point (a,b) of its domain then show that
 - $f(x,y)$ is continuous at (a,b)
 - f_x and f_y exist at (a,b)
- State and prove sufficient condition for differentiability of the function $f(x,y)$.
- State and prove Schwarz's theorem for equality of f_{xy} and f_{yx} at a point (a,b)
- State and prove Young's theorem for equality of f_{xy} and f_{yx} at a point (a,b)
- Evaluate

$$\text{Lim}_{(x,y) \rightarrow (0,0)} \frac{x^2(x+y)}{x^2+y^2}$$

$(x,y) \rightarrow (0,0)$

- 6) Show that the limit

$$\text{Lim}_{(x,y) \rightarrow (0,0)} \frac{\tan(x^2+y)}{x+y} \text{ does not exist.}$$

$(x,y) \rightarrow (0,0)$

- 7) Evaluate

$$\text{Lim}_{(x,y) \rightarrow (0,0)} \frac{x^2y^4}{(x^2+y^4)^2}$$

$(x,y) \rightarrow (0,0)$

- 8) Discuss the continuity of the function f defined by

$$f(x,y) = \frac{x^3+y^3}{x-y} \text{ when } x \neq y \text{ and}$$

$$= 0 \text{ for } x = y$$

at $(0,0)$

- 9) Show that the function $f(x,y)$ is continuous at $(0,0)$

$$\text{where } f(x,y) = \frac{xy(x^2-y^2)}{(x^2+y^2)} \text{ for } (x,y) \neq (0,0)$$

$$= 0 \text{ for } (x,y)=(0,0)$$

10) Examine the continuity of the function $f(x,y)$ at $(0,0)$

Where

$$f(x,y) = \frac{xy \sin \sqrt{x^2 + y^2}}{x^2 + y^2} \quad \text{for } (x,y) \neq (0,0)$$
$$= 0 \quad \text{for } x=y$$

at $(0,0)$

11) Show that the function $f(x,y)$ is continuous at $(0,0)$

Where

$$f(x,y) = y + x \sin \frac{1}{y} \quad \text{when } y \neq 0$$

x may or may not be 0.

$$= 0 \quad \text{when } y = 0$$

12) Discuss the continuity of the function $f(x,y)$ at $(0,0)$

Where

$$f(x,y) = \frac{x^2 y^2}{x^2 y^2 + (x-y)^2} \quad \text{for } (x,y) \neq (0,0)$$
$$= 0 \quad \text{for } (x,y) = (0,0).$$

13) If $u = \log (\tan x + \tan y + \tan z)$

Prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

14) If $u = x^2 \tan^{-1} \frac{y}{x} - y \tan^{-1} \frac{x}{y}$

Find $u_{xy}(x,y)$

15) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, show that $\nabla^2 u = 0$ where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

16) Let f be a function defined by

$$f(x,y) = \frac{x^2 y^2}{x^2 + y^2} \quad \text{for } (x,y) \neq (0,0)$$

and $f(0,0) = 0$

show that $f_{xy}(x,y)$ and $f_{yx}(x,y)$ are equal for $(x,y) \neq (0,0)$

and also show that they are not continuous at $(0,0)$.

17) For the example 16 prove that $f_{xy}(0,0) = f_{yx}(0,0)$

18) If $f(x,y) = \frac{x^2 y^2 (x^2 - y^2)}{x^2 + y^2}$ for $x^2 + y^2 \neq 0$ and $f(0,0) = 0$

Show that $f_{xy}(0,0) = f_{yx}(0,0)$

19) Let a function f be defined as,

$$f(x,y) = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y} \text{ when } xy \neq 0$$

and $f(x,y) = 0$ when $xy = 0$

show that, $f_{xy}(0,0) = 1$ and $f_{yx}(0,0) = -1$

20) Let f be function defined by,

$$f(x,y) = \frac{xy}{\sqrt{x^2 + y^2}} \text{ if } (x,y) \neq (0,0)$$

$$f(0,0) = 0$$

show that $f(x,y)$ is not differentiable at $(0,0)$.

21) Show that the function f defined by,

$$f(x,y) = \frac{x^4 + y^4}{x^2 + y^2} \text{ when } x^2 + y^2 \neq 0$$

$$= 0 \text{ when } x^2 + y^2 = 0$$

is differentiable at $(0,0)$.

22) Show that the function $f(x,y) = |x| + |y| \quad \forall x,y \in \mathbb{R}$

is continuous at the origin but not differentiable there.

23) Discuss the differentiability of a function f , where

$$f(x,y) = \frac{xy}{x^2 + y^2} \text{ when } (x,y) \neq (0,0)$$

$$f(0,0) = 0 \text{ at the origin.}$$

24) Let a function f be defined by,

$$f(x,y) = \frac{xy(x^2 - y^2)}{x^2 + y^2} \text{ when } (x,y) \neq (0,0)$$

$$= 0 \text{ when } x = 0 = y$$

Show that, $f_x(0,0) = 0 = f_y(0,0)$

Also show that,

$$f_x(x,y) = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2 + y^2)^2} \text{ for } (x,y) \neq (0,0)$$

25) Prove that the function defined in example 24 is differentiable at $(0,0)$.

26) Using differentials find an approximate value of $(2.01)(3.02)^2$

27) Estimate $f(1.02, 1.97)$, where $f(x,y) = \sqrt{x^2 + y^2}$

28) Find approximate value of $(3.9)^2(2.05) + (2.05)^3$

29) Find approximately the value of $(5.12)^2(6.85) - 3(6.85)$

30) Find approximate value of, $\sqrt{(3.012)^2 + (3.977)^2}$

UNIT -II

(Composite Function and Mean Value Theorem)

I) Objective questions (2 marks each)

1) If $u = f(x,y)$ $x = \Phi(t)$, $y = \Psi(t)$

Then state the formula for $\frac{du}{dt}$

2) If $w = f(u,v)$, $u = \Phi(x, y)$, $v = \Psi(x, y)$

Then state the formula for $\frac{\partial w}{\partial x}$.

3) If $w = f(u,v)$, $u = \Phi(x, y)$, $v = \Psi(x, y)$

Then state the formula for $\frac{\partial w}{\partial y}$

4) Define homogenous function of x and y with degree n .

5) State Euler's theorem for the homogenous function $f(x,y)$ with degree n .

6) If $u = G^{-1}\left[x^n f\left(\frac{y}{x}\right)\right]$ and $G^1(u) \neq 0$

Then what is the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

7) If $u = f(x,y)$ is a homogenous function of degree n then what is the value of

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$$

8) Using Mean value theorem complete the equality

$$f(a+h, b+k) = \dots + h f_x(a+\theta h, b+\theta k) + k f_y(a+\theta h, b+\theta k)$$

9) If $f(x,y) = a x^2 + 2hxy + by^2$ then find the degree of the homogeneous function using definition.

10) If $f(x,y) = x^4 + y^4$ then find the value of $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$

11) If $f(x,y) = (x^2 + y^2)^{\frac{1}{2}}$ then find the value of $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$

12) If $f(x,y) = x^2 + y^2$ then find the value of

$$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy}$$

13) If $z = x^2 + y^2$, $x = t^2 + 2y = 2t$

find $\frac{dz}{dt}$

14) If $z = \sin(x+y)$, $x = t^2 + 1$, $y = t^2$

find $\frac{dz}{dt}$

15) If $z = u^2 + v^2$, $u = x+y$, $v = x-y$

Find $\frac{\partial z}{\partial x}$

16) If $z = u^2 + v^2$, $u = x+y$, $v = x-y$

Find $\frac{\partial z}{\partial y}$

II) Multiple choice questions (1 marks each)

Choose the correct option from the given four options.

1) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$

Then $xu_x + yu_y = \dots\dots\dots$

- a) $\cos 2u$
- c) $\sin 2u$

- b) $\sin u$
- d) None of these.

- 2) If $f(x,y) = (x^2 + y^2)^{-1}$ then $xf'_x + yf'_y$ is
 a) 0
 b) $-f(x,y)$
 c) $f(x,y)$
 d) None of these.

3) If $z = x+y$, $x = 2t$, $y = 3t$.

then $\frac{dz}{dt}$ is

- a) 0
 b) 3
 c) 1
 d) 5

4) If $z=xy f\left(\frac{x}{y}\right)$ then

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \text{-----}$$

- a) z
 b) 0
 c) $\frac{1}{z}$
 d) $2z$.

II) Theory and Examples (3 – marks each)

1) If $u = f(x,y)$ is a differentiable function of x and y ,

$x = \Phi(t)$, $y = \Psi(t)$ are differentiable functions of t then prove that

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$$

2) If $w = f(u,v)$ is a differentiable function and $u = \Phi(x, y)$, $v = \Psi(x, y)$ are differentiable functions then prove that,

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \frac{\partial v}{\partial x}$$

3) If $w = f(u,v)$ is a differentiable function and $u = \Phi(x, y)$, $v = \Psi(x, y)$ are differentiable functions then prove that,

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial w}{\partial v} \frac{\partial v}{\partial y}$$

4) State and prove Euler's theorem for homogenous function in x and y .

5) If, $u = G^{-1}\left\{x^n f\left(\frac{y}{x}\right)\right\}$ and $G^1(u) \neq 0$ Then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{G(u)}{G^1(u)}$$
 Hence find, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ if $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$

6) If $u = f(x,y)$ is homogenous function of degree n in x and y then prove that,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

7) State and prove mean value theorem for the function $f(x,y)$.

8) Find $\frac{du}{dt}$ if $u = x^3 + y^3$, where $x = a \cos t$, $y = b \sin t$

9) Find $\frac{dz}{dt}$ if $z = xy^2 + x^2y$

Where, $x = at^2$, $y = 2at$

10) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$

Prove that, $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

11) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$

12) If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

13) If z is a function of x and y where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$

Show that, $\frac{\partial z}{\partial u} - \frac{\partial u}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$

14) Let $z = f(u, v)$ where $u = 2x-3y$ and $v = x+2y$, prove that,

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3 \frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$$

15) If $u = \tan^{-1} \frac{y}{x}$ where $x = e^t - e^{-t}$, $y = e^t + e^{-t}$, find $\frac{du}{dt}$

16) If $z = f(x, y) = \tan^{-1} \frac{x}{y}$, $x = u+v$, $y = u-v$, show that

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u-v}{u^2+v^2}$$

17) If $z = f(x, y)$, $x = uv$, $y = \frac{u+v}{u-v}$ then prove that,

$$2x \frac{\partial z}{\partial x} = u \frac{\partial z}{\partial u} + v \frac{\partial z}{\partial v}$$

18) If $z = f(u, v)$, $u = x^2 - y^2 - 2xy$, $v = y$ prove that the equation

$$(x+y) \frac{\partial z}{\partial x} + (x-y) \frac{\partial z}{\partial y} = 0 \text{ is equivalent to } \frac{\partial z}{\partial v} = 0$$

19) Verify Euler's Theorem for function, $f(x, y) = x^3 + y^3 - 3x^2y$

20) If $u = \operatorname{cosec}^{-1} \left[\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}} \right]^{\frac{1}{2}}$ show $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan u \left[\frac{13}{12} + \frac{\tan^2 u}{12} \right]$

21) If $u = \sin^{-1} \left[\frac{x^2 + 2xy}{\sqrt{x-y}} \right]^{\frac{1}{5}}$ find the values of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

22) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ prove that,

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \text{ and } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3$$

23) If $u = \sin^{-1} \sqrt{x+y^2}$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u$

24) If $u = \Phi\left(\frac{x}{y}\right) + f\left(\frac{y}{x}\right)$ then prove that, $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$

25) If $u = \sin^{-1}(x^2 + y^2)^{\frac{1}{5}}$, prove that,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{2}{25} \tan u (2 \tan^2 u - 3)$$

26) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ show that

i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u (1 - 4 \sin^2 u)$

27) Verify Euler's theorem for the function $f(x,y) = x^4 \log \frac{y}{x}$

28) If $f(x,y) = x^3 - xy^2$ show that θ used in the mean value theorem applied to the points (2,1) and (4,1) satisfies the quadratic equation $3\theta^2 + 6\theta - 4 = 0$

29) If $f(x,y) = x^2 y + 2xy^2$ show that the value of θ in the expression of the mean value theorem applied to the line segment joining the point (1,2) to (3,3) satisfies the equation $12\theta^2 + 30\theta - 19 = 0$

30) If $u = \tan^{-1} \frac{y}{x}$ show that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$

IV) Compulsory Examples (2 marks each)

1) If $z = f(x,y)$ where, $x = r \cos \theta$, $y = r \sin \theta$ then prove that,

$$\frac{\partial z}{\partial r} = \cos \theta \frac{\partial z}{\partial x} + \sin \theta \frac{\partial z}{\partial y}$$

2) If $z = f(x,y)$ where $x = r \cos \theta$, $y = r \sin \theta$ then prove that

$$\frac{\partial z}{\partial \theta} = -r \sin \theta \frac{\partial z}{\partial x} + r \cos \theta \frac{\partial z}{\partial y}$$

3) Verify Euler's theorem for the function $f(x,y) = \tan^{-1} \frac{y}{x}$

4) Verify Euler's theorem for the function $f(x,y) = (x^2 + y^2)^{\frac{1}{2}}$

5) If $u = \tan^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$

6) If $u = \sin^{-1} (x^2 + y^2)^{\frac{1}{5}}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{2}{5} \tan u$

7) If $u = x+y$ and $x = e^t$, $y = t^3$ then find $\frac{du}{dt}$ at $t=1$

8) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

Unit – III (Taylor's Theorem and Extreme values)

I) Objective questions (2 marks each)

- 1) Write the simplified mathematical Taylor's expansion for $f(x,y)$
- 2) Write the simplified mathematical Maclaurin's expansion for $f(x,y)$.
- 3) Define absolute maximum of the function $f(x,y)$ at (a,b)
- 4) Define absolute minimum of the function $f(x,y)$ at (a,b)

- 5) State the necessary condition for the extreme value.
- 6) Write the condition for critical point (a,b) to become a function f(x,y) maximum.
- 7) Write the condition for critical point (a,b) to become function f(x,y) minimum.
- 8) Under what condition the critical point (a,b) will be saddle point ?

9) prove that, $e^{x+y} = 1 + (x+y) + \frac{(x+y)^2}{2!} + \dots$

10) State the working rule to determine extreme value of the function f(x,y)

11) prove that

$$\sin(x+y) = (x+y) - \frac{(x+y)^3}{3!} - \dots$$

12) prove that $\cos(x+y) = 1 - \frac{(x+y)^2}{2!} + \frac{(x+y)^4}{4!} - \dots$

13) Find the Critical point for $f(x,y) = xy + \frac{50}{x} + \frac{20}{y}$

14) Find the Stationary points for $f(x,y) = x^3 y^2 (1-x-y)$

15) Find the Stationary points for $f(x,y) = x^3 + y^3 - 3axy$

16) Find the minimum value of $f(x,y) = 1 + x^2 + y^2$

II) Multiple Choice questions (1 mark each)

Choose the correct option from the given four options :

- 1) If $f(x,y) = x^2 - y^2 + 4$ then f has extreme value at - - - -
 - a) (1,1)
 - b) (0,0)
 - c) (1,0)
 - d) None of these.
- 2) To expand $x^2 + 2xy + y^3$ in powers of x -4 and y+2 by Taylor's theorem the values of h & k are - - - - .respectively.
 - a) 4 , 2
 - b) -4 , 2
 - c) 4 , -2
 - d) None of these.
- 3) If (a,b) is the Stationary point of the function f(x,y) and $f_{xx}(a,b) > 0, f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2 < 0$ then f(a,b) is,
 - a) Minimum
 - b) Saddle point
 - c) Maximum
 - d) None of these
- 4) If (a,b) is the critical point of the function f(x,y) and $f_{xx}(a,b) > 0, f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2 > 0$ then f(a,b) is,
 - a) Maximum
 - b) Saddle point
 - c) Minimum
 - d) None of these

III) Theory and Examples.(4 marks each)

- 1) State and prove Taylor's theorem for f(x,y)
- 2) State Taylor's theorem and hence obtain Macluarin's expansion in simplified form
- 3) State and prove necessary condition for extreme values of the function f(x,y)
- 4) State and prove the sufficient condition for the maximum value of the function f(x,y)
- 5) State and prove the sufficient condition for the minimum value of the function f(x,y)
- 6) Explain the Lagrange's method of undetermined multipliers to find extreme values of the function f(x,y,z).

7) Prove that $e^{ax} \sin by = by + abxy + \frac{e^u}{6} [(a^3 x^3 - 3ab^2 xy^2) \sin u + (3a^2 bx^2 y - b^3 y^3) \cos v]$

Where $u = a\theta x, v = b\theta y$

8) Expand $\sin xy$ in powers of $(x-1)$ and $(y - \frac{\pi}{2})$ upto and including second degree term.

9) Prove that $e^{x+y} = 1 + (x+y) + \frac{1}{2}(x+y)^2 + \frac{1}{6}(x+y)^3 e^{\theta x + \theta y}$

10) Show that for $0 < \theta < 1$

$$\sin x \sin y = xy - \frac{1}{6} [(x^3 + 3xy^2) \cos \theta x \sin \theta y + (y^3 + 3xy^2) \sin \theta x \cos \theta y]$$

11) Expand $x^3 + y^3 + xy^2$ in powers of $(x-1)$ and $(y-2)$

12) Expand $x^2 y$ as polynomial in $(x-1)$ and $(y+2)$ by using Taylor's theorem

13) Expand the function $f(x,y) = x^2 + xy - y^2$ by Taylor's theorem in powers of $(x-1)$ and $(y+2)$.

14) Expand $x^2 y + 3y - 2$ in powers of $(x-1)$ and $(y-2)$

15) Expand $e^{2x} \cos y$ as a Taylor's series about $(0,0)$ up to first three terms.

16) Write down the Taylor's expansion of $e^{ax} \cos by$ about $(0,0)$ up to and including terms of the second degree.

17) Expand $e^{ax} \log(1+y)$ in powers of x and y up to terms of third degree.

18) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree.

19) Find extreme values of $f(x,y) = x^3 y^2 (12 - 3x - 4y)$

20) Find extreme values of $f(x,y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$

21) Find the Stationary points and determine the nature of the function

$$f(x,y) = x^3 + y^3 - 3x - 12y + 20$$

22) Find the least value of the function, $f(x,y) = xy + \frac{50}{x} + \frac{50}{y}$

23) Investigate the maximum and minimum values of

$$f(x,y) = 3x^2 y - 3x^2 - 3y^2 + y^3 + 2$$

24) Discuss the maximum and minimum of the function $u = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

25) Find the rectangle of perimeter 12cm which has maximum area.

26) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

27) Determine the minimum distance from origin to the plane

$$3x + 2y + z - 12 = 0$$

28) Find the minimum and maximum distance from the origin to the curve $5x^2 + 6xy + 5y^2 = 8$

29) If a, b, c are +ve numbers find the extreme value of $f(x,y,z) = x^a y^b z^c$ subject to the condition $x+y+z=1$

30) Find extreme values of $f(x,y) = xy(a-x-y)$

Unit IV (Double and Triple Integrals)

I) Objective questions (2 marks each)

1) Define double integral $\iint_R f(x,y) dR$

2) Evaluate : $\int_0^2 \int_0^x (x+2y) dy dx$

3) Evaluate : $\int_1^2 \int_0^1 (x^2 + y^2) dx dy$

4) Evaluate : $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dx dy$

5) Evaluate : $\int_1^2 \int_0^x \frac{dx dy}{x^2 + y^2}$

6) Evaluate : $\int_0^a \int_0^b (x^2 + y^2) dx dy$

7) Evaluate : $\int_0^4 \int_0^{\sqrt{y}} xy dx dy$

8) Evaluate : $\iint xy dx dy$ over the rectangle bounded by $x=2, x=5, y=1, y=2$.

9) Evaluate : $\int_0^1 dx \int_0^1 (x + y) dy$

10) Evaluate : $\int_0^1 \int_0^{1-x} xy dy dx$

11) Define triple integral $\iiint_v f(x, y, z) dv$

12) Evaluate : $\int_0^1 \int_0^1 \int_0^1 (x + y + z) dx dy dz$

13) Evaluate ; $\int_0^1 \int_0^2 \int_1^2 xyz dx dy dz$

14) Evaluate : $\int_3^4 \left[\int_1^2 \frac{dy}{(x+y)^2} \right] dx$

15) Evaluate : $\int_1^2 \int_1^2 (x - y) dx dy$

II) Multiple Choice questions (1 marks each)

Choose the correct option from the given options.

1) The area of region bounded by the circle $x^2 + y^2 = a^2$ is

- a) $\Pi^2 a$ unit b) Πa^2 unit²
 c) Πa^3 unit³ d) Πa unit

2) The area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

- a) 12Π unit² b) 12 unit
 c) 12Π unit³ d) 12Π

3) The volume of the sphere $x^2 + y^2 + z^2 = a^2$ is

- a) Πa^2 unit² b) Πa^3 unit
 c) $\frac{4}{3} \Pi a^3$ unit³ d) Πa unit

4) The Volume of ellipsoid $\frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1$ is

- a) $\frac{4}{3}\Pi$ unit b) $\frac{4}{3}\Pi^2$ unit²
 c) $\frac{24}{3}\Pi$ unit³ d) $\frac{12}{3}\Pi$ unit³

III) Theory and Examples (6 marks each)

1) a) Draw a sketch of the region of integration

$$\int_{-1}^2 \int_{x^2}^{x+2} f(x, y) dx dy$$

b) Evaluate : $\int_{y=0}^3 \int_{x=0}^2 \int_{z=0}^1 (x + y + z) dz dx dy$

2) a) Draw a sketch of the region of integration , $\int_0^4 dx \int_0^{\sqrt{25-x^2}} f(x, y) dy$

c) Evaluate : $\int_{x=0}^1 \int_{y=0}^1 \int_{z=1}^2 x^2 yz dz dy dx$

3) a) Draw a sketch of the region of integration $\int_0^2 dy \int_y^{4-y} f(x, y) dx$

b) Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz dx dy dz$

4) a) Draw a sketch of the region of integration $\int_1^2 dx \int_{\frac{x}{2}}^{2x} f(x, y) dy$

b) Evaluate $\iiint (x + y + z) dx dy dz$ over the tetrahedron $x=0$, $y=0$, $z=0$ and $x+y+z=1$

5) a) Change the order of integration and hence evaluate

$$\int_0^a \int_y^a \frac{xdx dy}{x^2 + y^2}$$

b) Evaluate : $\int_0^a \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$

6) a) Evaluate : $\iint xy dx dy$ over the region in the positive quadrant for which $x+y \leq 1$

b) Evaluate : $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$

7) a) Using double integral find the area of the circle $x^2 + y^2 = a^2$

b) Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$

8) a) Change the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$

b) Evaluate $\iiint \frac{dx dy dz}{(x + y + z + 1)^3}$ over the region $x \geq 0, y \geq 0, z \geq 0, x + y + z \geq 1$

9) a) Change the order of integration $\int_1^{2x} \int_x^{2x} f(x, y) dx dy$

b) Evaluate ; $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dx dy dz$

10) a) Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using double integration.

b) Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{(x+y)^2} x dx dy dz$

11) a) Using double integration find the area of the region bounded by the parabola's $y^2 = 4x$ and $x^2 = 4y$

b) Find the volume of the sphere of radius 5.

12) a) Evaluate $\iint e^{2x+3y} dx dy$ over the triangle bounded by $x=0, y=0$, and $x+y=1$

b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

13) a) Evaluate $\iint y dx dy$ over the area bounded by $y = x^2$ and $x+y=2$

b) Find the volume of the region bounded by $x=0, y=0, z=0$ and $x+y+z=1$

14) a) Evaluate : $\iint x^2 y^2 dx dy$ over the region $x^2 + y^2 \leq 1$

b) Using triple integration find the volume of the sphere of radius a.

15) a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx dy}{(1+e^y)\sqrt{1-x^2-y^2}}$

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y+z=3, z=0$

III) Compulsory Examples (2 marks each)

1) Find the area of square bounded by lines $x=0, x=1, y=0, y=1$ using double integration.

2) Evaluate $\int_{x=0}^2 \int_{y=0}^x \int_{z=0}^y e^p dx dy dz$, where p is a real.

3) Find the area bounded by the lines $x=0, x=a, y=0, y=b$.

4) Find the volume of region bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$.

5) Find the area of the circle of radius 1 using double integration.

6) Evaluate $\int_2^1 \int_0^2 x^3 y^3 dx dy$

7) Evaluate : $\int_0^1 \int_0^1 (x+y) dx dy$

8) Evaluate: $\int_a^b \int_a^b xy dx dy$