Mathematics Paper-III (B)

Unit –I

Set, Proposition, Computability and Languages

Marks 02

1. Define a) Finite Set

b) Infinite Set

c) Successor of a Set, Give one example of it.

2. Define Cardinality of a Set and give one example of it.

3. Define Countable infinite Set and give one example of it.

4. Define Uncountable infinite Set and give one example of it.

5. State the Principle of inclusion and exclusion for two sets

6. State the Principle of inclusion and exclusion for three sets

7. State the Principle of inclusion and exclusion for four sets

8. State the Principle of inclusion and exclusion for n sets

9. State the Principle of Mathematical Induction.

10. State the Generalized Principle of Mathematical Induction.

11. Define Ordered Set .Give examples of it.

12. Define Language .Give Examples of it.

13. Define Type 0-grammar.

14. Define Type 1-grammar.

15. Define Type 2-grammar.

16. Define Type 3-grammar.

17. Define Type i-Language

18. Choose Proper answer of each of the following

1) The Cardinality of empty Set is ---------

A) 0 B) 1 C) N_0 D) ϕ

2) The Cardinality of infinite Set is ---------

A) 0 B) ∞ C) N_0 D) None of these

3) A countable infinite Set is also called as --------

A) denumerable B) Non-denumerable

C) Uncountable D) None of these

4) The Set of positive rational numbers is ----------

A) Uncountable B) Denumerable

C) Empty set D) None of these

5) Cardinality of two disjoint Set A and B is -----------------

A) $|A \cap B|$ B) $|A \cup B|$ C) $|A| + |B|$ D) $|A - B|$

6) If A is subset of the universal set U, then $|A'|$ is ---------

A) $|A| - |U|$ B) $|A + U|$ C) $|U| - |A|$ D) $|A - U|$

7) For any two sets A and B Cardinality of $A \cup B$ is less than equal to -----

A) $|A| - |B|$ B) $|A| + |B|$ C) $|B| - |A|$ D) $2|A \cap B|$

8) The Cardinality of Symmetric difference of two sets A and B is ---------

A) $|A| + |B| - |A \cap B|$ B) $|A| + |B| - 2|A \cap B|$

C) $|A| - |B| + |A \cap B|$ D) $|A| + |B| - 3|A \cap B|$

9) Languages are defined as a set of --------

A) String B) Word C) n-type D) Alphabet

10) The process of generating a sentence is called as -------

A) Description B) Derivation C) Production D) Grammar.

11) A type 3 or type 2 grammar is also a trivially ------------ grammar.

A) Type-0 B) Type -2 C) Type -3 D) Type -1

12) A type –i Language can be specified by a ----------

A) Type- i grammar B) Type –i-1 grammar

C) Type -2 grammars D) None of these

13) There are Languages that can not be specified by -------- grammar.

A) Language B) Phrase Structure C) Structure D) Phrase

14) All programming Language are almost type -------- language.

A) 0 B) 1 C) 2 D) 3

15) A phrase Structure grammar having no restriction or production is -------

A) 0 B) 1 C) 2 D) 3

16) The Successor of a set $\{\phi\{\phi\}\}$ is

A) $\{\phi\}$ B) $\{\phi, \{\phi\}\}$ C) $\{\phi, \{\phi\}, \{\phi, \phi\}\}$ D) $\{\phi, \{\phi\}, \{\phi, \phi\}, \{\phi, \phi, \phi\}\}$

19. Find the successor of a following sets

1) {a, b} 2) ϕ 3) $\{\phi, \{\phi\}\}$ 4) {1, 2, 3}

20. Show that set of Natural number is countable infinite set.

21. Show that set {0, 1, 2, ------} is countable infinite.

22. Show that set of even positive integers is countable infinite.

23. Prove that for any two sets A and B, $|A \cap B| \le \min\{|A|, |B|\}$

24. Prove that $|A \cup B| \le |A| + |B|$

25. Prove that $|A - B| \ge |A| - |B|$

26. If A is subset of universal set U,then Prove that $|A'| = |U| - |A|$

27. Find the Cardinality of the symmetric difference of two sets A and B.

28. If A= Set of integers between 1 to 200 that are divisible by 3 and B = set of integers between 1 to 200 that are divisible by 7 .Find $|A|$ and $|B|$.

29. If A and B are the set of integers between 1 to 250 that are divisible by 2 and 3 respectively, then find $|A|$, $|B|$ and $|A \cap B|$.

30. If A and B are set of multiples of 2 and 3 respectively, then show that $|A| = |B|$ and $|A \cup B|$.

31. If $|A| = 40$, $|B| = 60$ and $|A \cap B| = 30$, $|U| = 200$ then find $|A \cup B|$, $|A'|$.

32. Let $P(n): n^3 + 2n$ is divisible by 3, for all $n \ge 1$. Prove that P (1), P (2) are true.

33. Show that P(2) and P(3) are true if $n^4 - 4n^2$ is divisible by 3, for all $n \ge 2$

34. If P (n) is $n^4 + n^3 - 2n^2$ is even .Show that P (2) and P (4) are true.

35. If P (n) is $n^2 - 1$ is divisible by 24 if n is odd positive integer greater than one, Show that P (2) and P (3) are true.

36. If P (n) is $x^n - y^n$ is divisible by x +y, for even positive integer, show that P (2) and P (4) are true.

37. If P (n) is $n! > 2^n, \forall n \ge 4$. Show that P (4) is true.

38. If P (n) is $2^n > n^3, \forall n \ge 10$, Show that P (10) is true.

39. If P(n) is $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + - - - - + \frac{1}{\sqrt{n}} > \sqrt{n}$ for n >1, then Show that P(2) is true.

40. Determine a type of the grammar G which consists of the production.

41. N = {S,A,B}, T = {a, b } and S is the starting symbol and

$P = \{S \to aA, A \to aAB, B \to b, A \to a\}$

42. N = {S,A}, T = {a, b } and S is the Sarting symbol and

$P = \{S \to bS, S \to aA, A \to aS, A \to bS, A \to a, S \to b\}$

43. N = {S,A,B}T = {a, b } and S is the Sarting symbol and

$P = \{S \to BAB, S \to ABA, A \to AB, B \to BA, A \to SA, A \to ab, B \to b\}$

44. Let {A, B, C, S} be the set of non-terminals with S being starting Symbol. Let T = {a, b} be the set of terminals. For the each set of productions in the following determine the type of the corresponding grammar.

a) $\{S \to ABC, A \to a, A \to b, aB \to b, bB \to a, bC \to a, aC \to b\}$

b) $\{S \to AB, AB \to BA, A \to a, B \to b\}$

c) $\{S \to AB, S \to bA, A \to a, B \to b\}$

Marks - 04

1. Show that set of integers is countable infinite.
2. Show that set of all positive rational numbers is countable infinite set.
3. Show that the set $\left\{1,\frac{1}{2},\frac{1}{3},------\right\}$ is countable infinite. Find also cardinality of the set.
4. Show that set of rational is countable infinite.
5. Show that set if irrational is not countable infinite.
6. Show that the set $N \times N$ is countable infinite.
7. Determine the Cardinality of the sets
   a) A = { n^7 / n is positive integer }
   b) B = { n^{109} / n is positive integer }
   c) $A \cup B$
   d) $A \cap B$
8. Determine the Cardinality of the sets
   a) A = set of all integers multiple of 5.
   b) A = Set of all integer multiple of 7.

   Show that $|A| = |B|$. Find $|A \cup B|$ and $|A - B|$
9. If A and B are two sets,then Prove that $|A \cup B| = |A| + |B| - |A \cap B|$
10. Find integers between 1 to 100 that are divisible by 3 or 7.
11. Among the integers1 to 300, how many of them are not divisible by 3 not by 5 & not by 7?
12. A Departmental library of certain college has two dozen introductory test books on discrete Mathematics and is concerned with topics
    a) Graph Theory
    b) Boolean Algebra
    c) Finite state machines.

    The following data is the number of books that contain material or these topics.

    i) $|A| = 8$ ii) $|B| = 13$ iii) $|C| = 13$ iv) $|A \cap B| = 5$

    v) $|A \cap C| = 3$ vi) $|B \cap C| = 6$ vii) $|A \cap B \cap C| = 2$
13. At a Certain College, 60% of the Students play Cricket,

    50% of them play Chess, 70% of them play tennis,

    20% play Cricket and Chess, 30% play Cricket and tennis and

    40% play chess and tennis.

What is the percentage of teachers who play Cricket, Chess and tennis?

14. Let A, B, C be subsets of universal set U,

Suppose that $|U| = 200$, $|A \cap B \cap C| = 20$, $|A \cap B| = 30$, $|A \cap C| = 25$, $|B \cap C| = 35$,

$|A| = 40$, $|B| = 60$, $|C| = 70$.

Find $|A \cup B \cup C|$, $|A' \cap B'|$, $|A' \cap B' \cap C'|$

15. Among 100 Students,32 study Mathematics,

20 study Physics, 45 study Electronics,

15 study Mathematics and Electronics, 7 study mathematics and Physics,

10 study Physics and Electronic and 30 do not study any of the three subjects.

Find the number of student, Studying all three subjects .Find also the number of students studying exactly one of the three subjects.

16. In the department of Computer Science, there are 350 students, it is known that 92 can program in C+, 120 can program in Java, 51 can program in C, 12 in C+ and Java,18 in C and C+, 24 in C and Java and 6 in all three languages. Determine the number of students who can program in none of the languages mentioned.

17. Determine the number of integers between 1 to 250 that are divisible by 3, 5 and 7.

Prove that by induction,

18. $1+2+2^2+2^3+----+2^n = 2^{n+1}-1, n \in N$

19. $(11)^{n+2}+(12)^{2n+1}$ is divisible by 133 , $\forall n \geq 1$

20. $n > 2^n$, $for\ n \geq 4$

21. $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+----+\frac{1}{\sqrt{n}} > \sqrt{n}, \forall n > 1$

22. $2^n > n^3, \forall n \geq 10$

23. $n^4 - 4n^2$ is divisible by 3 , $\forall n \geq 2$

24. Sum of the cubes of three consecutive integers is divisible by 9.

25. $x^n - y^n$ is divisible by x + y if n is even positive integer.

26. $\frac{d}{dx} x^n = nx^{n-1}, \forall n \geq 1$

27. $7^{2n} + 16n - 1$ is divisible by 64, $\forall n \geq 1$

28. $5^n - 4n - 1$, is divisible by 16, $\forall n \geq 1$

29. (1) 1! + (2)2! + (3) 3! +---------+ (n) n! = (n+1)!-1

30. $1^3+2^3+3^3+------+n^3 = (1+2+---+n)^3, n \geq 2$

31. Show that $1^2+3^2+5^2+------+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}, \forall n\geq 1$

32. Show that $\frac{1}{1.3}+\frac{1}{3.5}+----+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}, \forall n\geq 1$

33. Show that $\frac{1}{1.2}+\frac{1}{2.3}+----+\frac{1}{n(n+1)}=\frac{n}{n+1}, \forall n\geq 1$

Construct Phrase Structure grammar for the language L Where L is

34. { aaaa,aabb,bbaa,bbbb}

35. $\left\{a^i b^{2i} / i\geq 1\right\}$

36. $\left\{x / x=a^n b^n,\ for\ n\geq 1\right\}$

37. Consider the phrase structure grammar G = { N, T, P, S }

Where N = {A, B, C), T = {a, b},

$P=\left\{S\to aS, B\to b, B\to bA, A\to aB\right\}$ And S is the staring symbol.. Describe the language in G.

38. Show that the language $L=\left\{a^n b^n c^n / n>1\right\}$ can be generated by G = {N, T, A, S}

Where N = {S, B, C}, T = {a, b, r}

$P=\left\{S\to aSBr, S\to aBc, rb\to BC, aB\to ab, bB\to bb, Br\to br, Cr\to rr\right\}$

and S is Starting symbol.

39. Find the language L, grammar G = { N, T, P, S }

Where N = {S, W}, T = {a, b, c}, $P=\left\{S\to aSb, Sb\to bw, abw\to c\right\}$

and S is the staring Symbol.

40. Find the language L, grammar G = { N, T, P, S }

Where N = {S, A, B}, T = {a, b, c}, $P=\left\{S\to AB, A\to ab, A\to aAb, B\to c, B\to BC\right\}$

and S is the Staring Symbol.

41. Determine the type of grammar in the following, where {A, B, C, S} is the set of non-terminals with S being starting symbol, {a, b} be set of terminals and set of productions is

a) $\left\{S\to ABC, A\to a, A\to b, aB\to b, bB\to a, bc\to a, ac\to b\right\}$

b) $\left\{S\to AB, AB\to BA, A\to a, B\to b\right\}$

c) $\left\{S\to AB, S\to BA, A\to a, B\to b\right\}$

d) $\left\{S\to aB, S\to BA, A\to a, B\to b\right\}$

42. Let {A, B, C, S} be the set of non-terminals with starting Symbol S. Let {a, b, r} be the set of terminals. Describe the language specified by each set of productions either verbally or in set theoretic notation

a) $\{S \to aA, S \to a, A \to ab\}$

b) $\{S \to abS, S \to aA, A \to a\}$

c) $\{S \to aAB, A \to aB, A \to a, B \to b, B \to c\}$

d) $\{S \to aSB, S \to aB, A \to b, B \to c\}$

e) $\{S \to Sa, S \to AB, A \to aA, A \to a, B \to b\}$

f) $\{S \to AB, A \to aA, A \to a, B \to Bb, B \to b\}$

g) $\{S \to AB, A \to ab, A \to aAb, B \to c, B \to Br\}$

h) $\{S \to aS, S \to bA, A \to aA, A \to a\}$

i) $\{S \to aA, A \to bA, A \to bc, C \to rC, C \to r\}$

j) $\{S \to BA, A \to Aa, A \to a, B \to Bb, B \to r\}$

k) $\{S \to AB, A \to aA, A \to b, B \to bB, B \to a\}$

Mathematics Paper-III (B)

Unit –II

Graphs

Theory Question (4 or 06 Marks)

1. Show that in any graph the number of odd vertices is always even.

2. Let G be a simple graph with n vertices .Show that G has at most $\frac{n(n-1)}{2}$ edges.

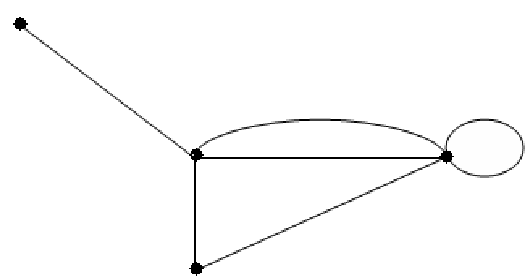
3. If G is a complete graph with n vertices then Show that G has exactly $\frac{n(n-1)}{2}$ edges.

4. Does there exist a regular graph of degree 5 on 7 vertices? Justify.

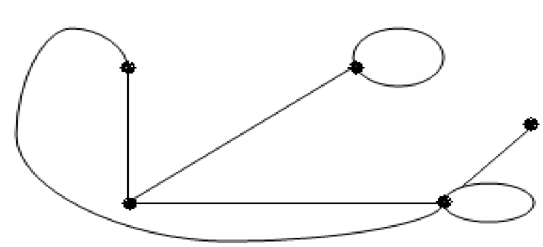
5. Does there exits a graph on 5 vertices whose degree are 1,2,3,4 and 5? Justify.

6. If G is a self complementary graph on n vertices then show that n = 4k or n = 4k+1.for some non-negative integer k.

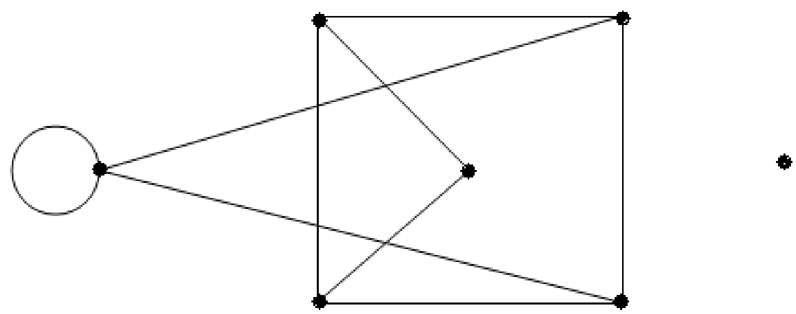
7. State Handshaking Lemma and verify it for following graph.



8. State Handshaking Lemma and verify it for the following graph.



9. Determine degree of each vertex in the following graph .Also verify Handshaking Lemma



10. Write a short note on “Konisberg’s seven bridges Problem”.

11. Write short note on “Travelling Salesman Problem”.

12. Explain “Dijkstra’s algorithm “to find Shortest Path.

13. If a simple graph has exactly two vertices of odd degree, then show that there is a path joining these two vertices.

14. Let G be a simple graph with K-components, n-vertices and m-edges. Prove that $m \geq n - k$.

15. Let G be a graph with an Eulerian path. Prove that G is connected with zero or two vertices of odd degree.

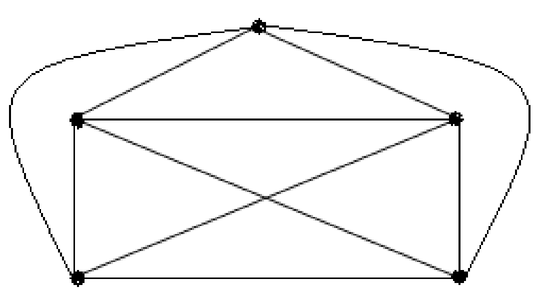
16. Let G be a connected simple and planar graph with p-vertices and q –edges. If $p \geq 3$ then show that $q \leq 3p - 6$.

17. Let G be a connected, simple and planar graph with p-vertices, q-edges and number circuits of length 3.If $p \geq 3$ then show that $q \leq 2p - 4$.

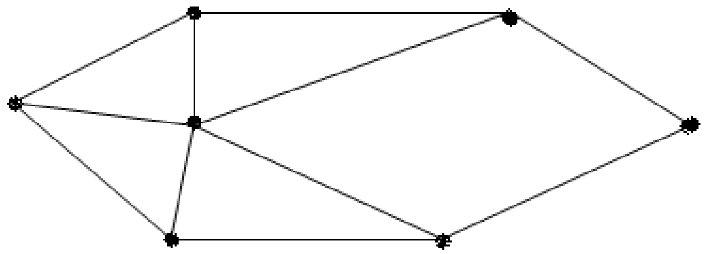
18. Show that the kuratowsk’s first graph K_5 is not a planar graph.

19. Show that the kuratowsk’s second graph $K_{3,3}$ is not a planar graph.

20. Using nearest neighbour method find the Hamiltonian cycle for the following graph.



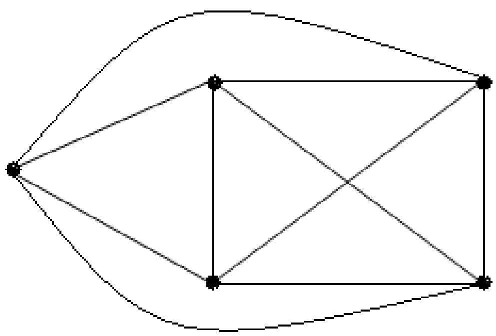
21. Find the shortest path between a and z in the following weight graph by Dijkstra’s algorithm.



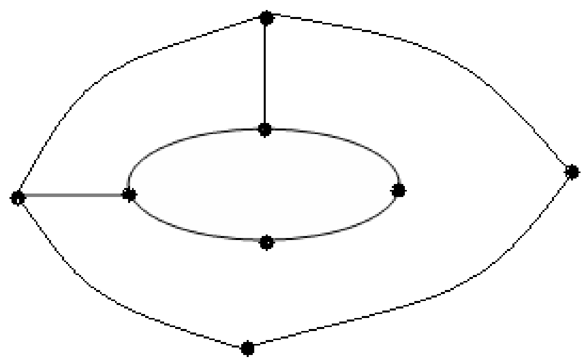
22. Using nearest neighbour method determine a Hamiltonian circuit for the following graph.

i) Starting from a.

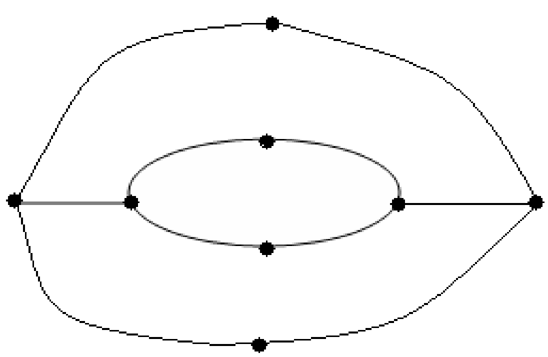
ii) Starting from d.



23. Are the following graphs isomorphic? Why?

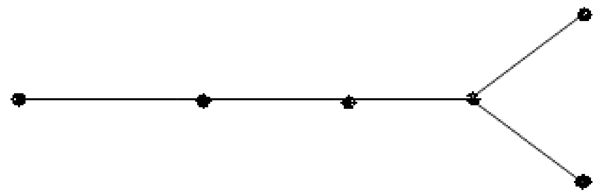


G_1

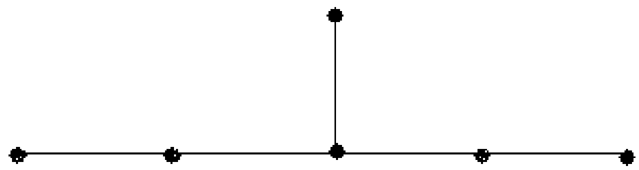


G_2

24. Are the following graphs isomorphic? Why?

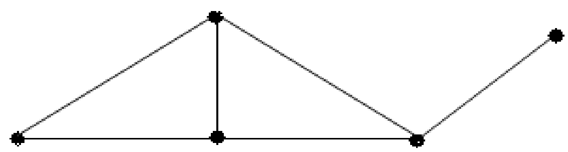


G_1

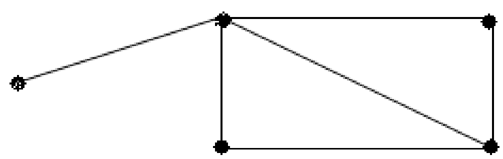


G_2

25. Are the following graphs isomorphic? Why?



G_1



G_2

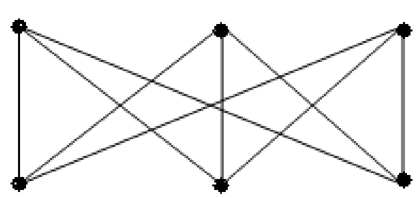
26. Let G be a graph with n-vertices. If these is a path from vertex u1 to vertex u2 in G then Show that there is a path of number more than n-1 edges from vertex u1 to vertex u2.

27. Find all self complementary graphs with 4 –vertices.

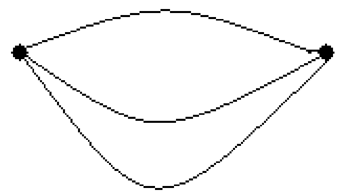
28. Find all self complementary graphs with 5 –vertices.

29. Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty sets v_1 and v_2 such that there exists numbers edges in G whose one end vertex is in v_1 and other is in $v_{2.}$

30. Define complement of a graph .Draw the complement of the following graph.



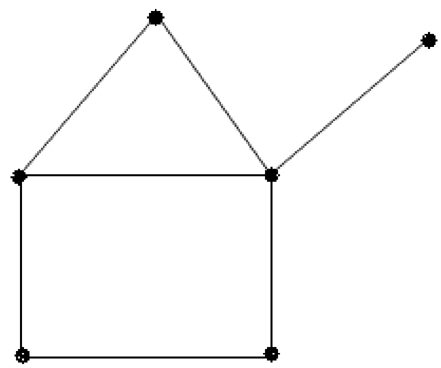
31. Find all sub graphs of



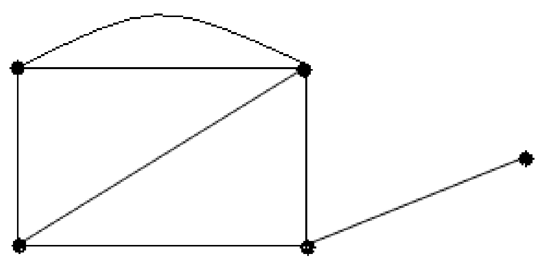
32. Draw i) $G_1 \cup G_2$

ii) $G_1 \cap G_2$

iii) $G_1 \oplus G_2$

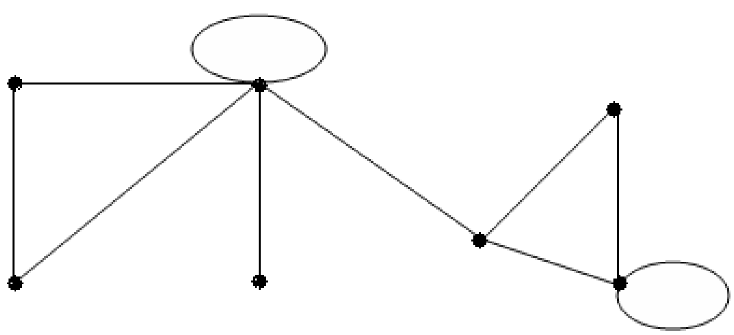


G_1



G_2

33. Draw i) $G - V_3$ ii) $G - e_3$

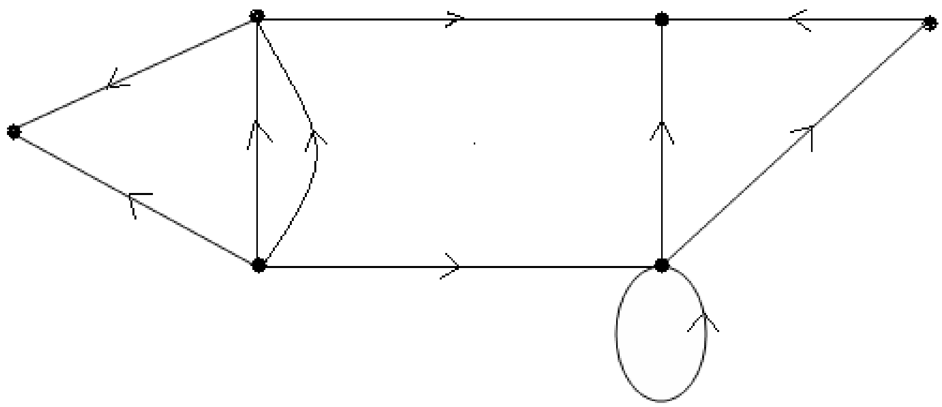


G

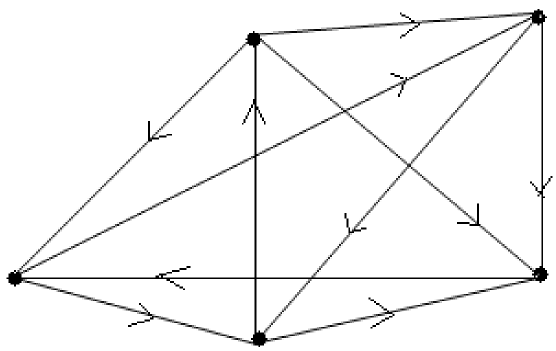
34. Prove that a graph with n-vertices and n-1 edges has either a vertex of degree or an isolated vertex.

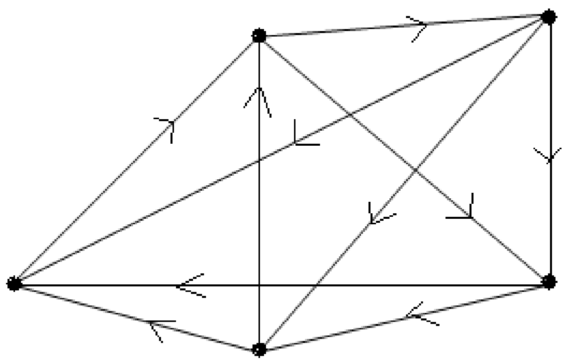
35. Let G be a graph with 6 vertices. If 2 vertices are of degree 4 and 4 vertices are of degree 2, and then determine the number of edges in G. Draw a graph.

36. Find indegree and outdegree of each vertex in the following diagraph and verify that $\sum d^{-}(v_i) = \sum d^{+}(v_i)$.



37. Show that the following diagraph are isomorphic.





38. By an example show that the sum of the squares of the indegree over all vertices is equal to the sum of the squares of the out degrees over all vertices in any directed complete graph.

2 Marks

1. Define i) a self loop ii) Parallel edges

2. Define i) a simple graph ii) a multiple graph

3. What is degree of a vertex?

4. Define i) An isolated vertex ii) A pendant vertex iii) An odd vertex iv) An Even vertex

5. State Handshaking Lemma.

6. Define i) A regular graph ii) A complete bipartite graph.

7. Define Isomorphism of graphs.

8. Define a planar graph .State Euler's formula for planar graph.

9. Define Eulerian and Hamiltonian graph.

10. Draw K_4 and $K_{5.}$

11. Draw $K_{2,3}$ and $K_{3,4.}$

12. Draw a complete graph on 6 vertices.

13. Explain the term: Fusion of vertices.

14. Define a ring sum of two graphs.

15. Define i) Union of two graphs ii) Intersection of two graphs

16. Define i) Bipartite graph ii) Complete bipartite graphs

17. Draw a regular graph of degree 2 on 6 vertices

18. The number of edges in K_5 are ------

A) 0 B) 5 C) 8 D) 10

19. The number of edges in K_n are -------

A) 0 B) n C) n-1 D) $\frac{n(n-1)}{2}$

20. The maximum number of edges in a simple graph on n vertices are -------

A) n B) n-1 C) $\frac{n(n+1)}{2}$ D) $\frac{n(n-1)}{2}$

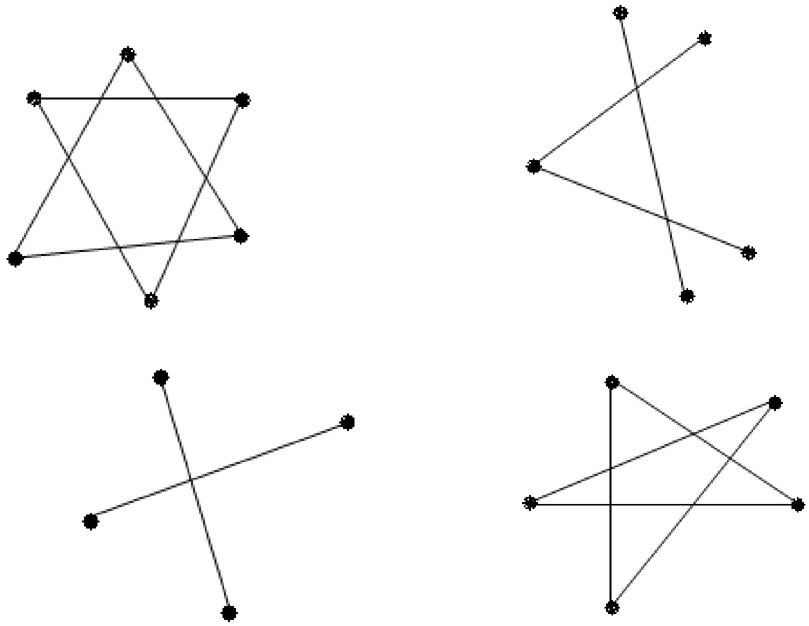
21. The graph gives below is ---------

A) Connected B) Disconnected C) Regular D) Null graph

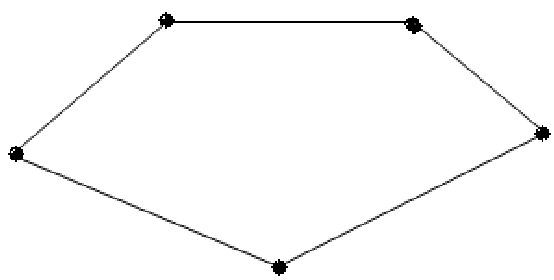
22. A graph G having 4 vertices with $d(v_1)=3, d(v_2)=2, d(v_3)=3, d(v_4)=2,$ has --------- number of edges

A) 4 B) 3 C) 5 D) 10

23. Identify the connected graph from the following graphs



24. Draw complement of the following graph.

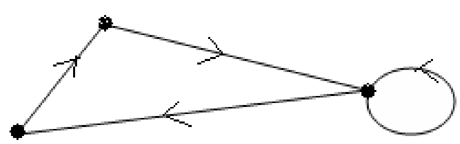


25. Define vertex connectivity and edge connectivity

26. If G is a balanced diagraph then-----------

A) $\sum d^{+}(v_i) = \sum d^{-}(v_i)$ B) $\sum d^{+}(v_i) > \sum d^{-}(v_i)$ C) $\sum d^{+}(v_i) < \sum d^{-}(v_i)$

27. The following graph G is -----------



A) Balanced but not regular B) Regular but not balanced

C) Regular and balanced D) Neither regular nor balanced

28. Define a complete diagraph.

29. Define i) A walk

ii)A path

iii) A cycle

30. Let G be a connected planar graph with p-vertices ,q-edges and r-faces, then p – q + r =

A) 3 B) 0 C) 2 D) None of these

31. Degree of each vertex in K_n is -----------

A) n B) n-1 C) n +1 D) None of these

32. Degree of each vertex in $K_{3,3}$is -------

A) 3 B) 6 C) 9 D) None of these

Mathematics Paper-III (B)

Unit-III

Finite State Machines

Marks -2

1. Define a) Finite State Machine

b) State Transition Function.

c) Output Function

2. Define State transition table and give example of it.

3. Define a) Equivalent Machine

b) Equivalent States

4. Explain 0-equivalent state and 1- equivalent state

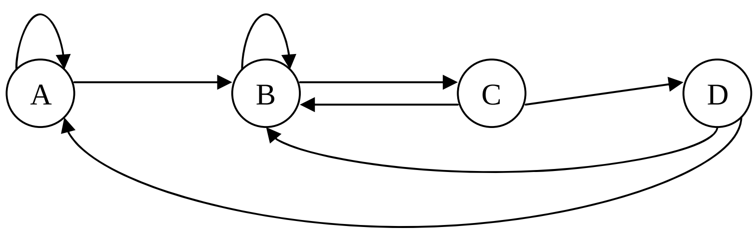
5. Define K- equivalent state

6. Explain finite state language

7. Explain finite state Machines as language recognizers.

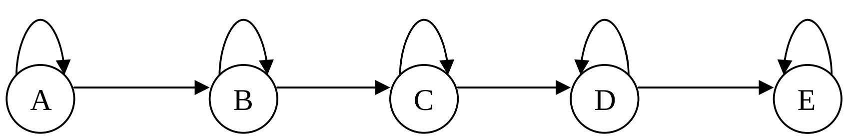
8. State Pumping Lemma for finite state languages in the literature.

9. A finite state machine is given by ,



Find its input symbols, its initial state and accepting state and transition table

10. A finite state machine is given by



Find its input symbols, its initial state and accepting state and transition table

11. Prove that every tree is a bipartite graph.

12. Find all trees having exactly three leaves and 6 vertices.

13. What is the sum of degree of all vertices in a tree with n-vertices?

14. A complete graph k_n is a tree determine the value of n.

15. Construct a tree with σ vertices having

a. Minimum number of pendant Vertices.

b. Maximum number of pendant Vertices.

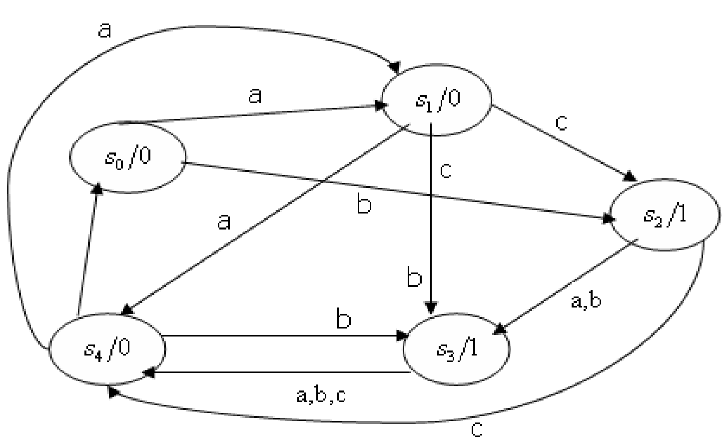
16. Describe the following finite state machine graphically.

State	Input		output
	1	2	
A	B	C	0
B	C	D	0
C	D	E	0
D	E	B	0
E	B	C	1

17. List all 0-equivalent states of the following machine.

State		A	B	C	D	E	F	G	H
Input	0	F	D	G	E	G	A	C	A
	1	B	C	B	A	A	G	H	H
Output		0	0	0	1	0	1	1	1

18. Construct the state transition table of the finite state machine given by the following diagraph.



19. Draw all non-isomorphic trees on 4 Vertices

20. State pumping Lemma for finite state Language in the literature.

Definitions

1. Define a) Tree

b) Forest

2. Define a) Pendant Vertex

b) Leaf

3. Define a) Trivial tree

b) Branch node

4. Define finite state machine.

5. When you can say that two finite state machines are equivalent.

6. When you can say that two states are equivalent.

7. When you can say that two states are 1-equivalent.

8. Define a) Accepting State

b) Rejecting State

9. Define finite State Language.

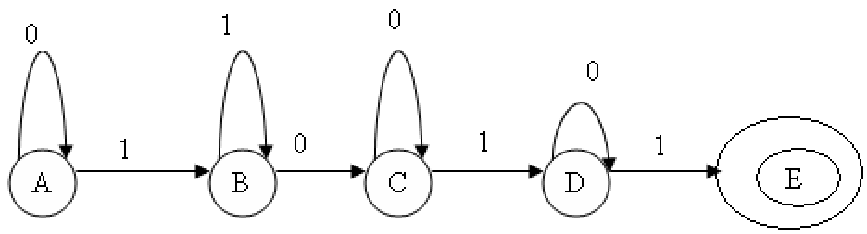
Numerical Examples –

1. Draw four non-isomorphic trees with 6-vertices.

2. A tree has two vertices of degree 2 ,one vertex of degree 3 and three vertices of degree 4. Determine number of vertices and edges occurring in the tree.

3. Draw a tree with 7 vertices.

4. A finite state machine is given by the following fig.



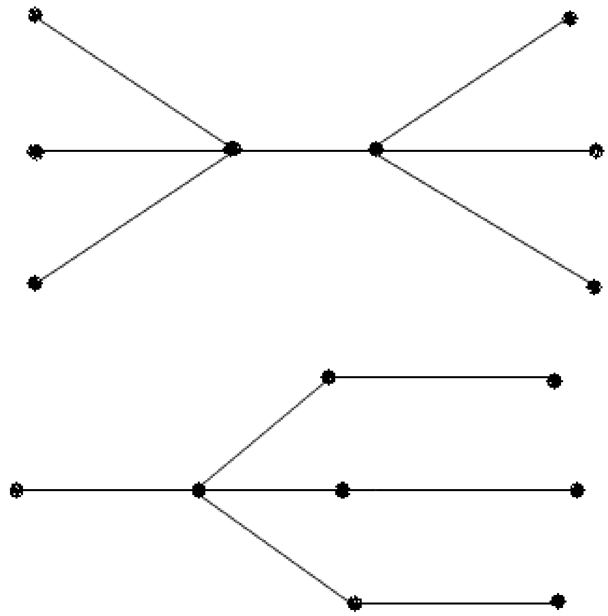
Find all states of it, initial state and accepting state.

5. For the finite state machine as shown in the table –

State.		A	B	C	D	E	F	G	H
Input	0	B	A	G	H	A	H	A	A
	1	F	F	A	B	G	C	D	C
Output		0	0	0	0	0	1	1	1

Find 1- equivalent states

6. Find the leaves and branch nodes of the following tree.



7. A tree has 2n pendant vertices, 3n vertices of degree 2 and n vertices of degree 3 .Determine numbers of vertices and edges in a tree.

8. A tree has 2 vertices of degree 2 one vertex of degree and three vertices of degree 4 .Determine numbers of the pendent vertices.

9. Find sum of degree of all vertices in a tree with 10 vertices.

True or False Justify

1. In a tree numbers of vertices is always even.
2. Numbers of edges in a tree is greater than numbers of vertices.
3. A tree with two or more vertices has at least two leaves.
4. If the state S_i and Sj are K-equivalent, then S_i and Sj are K-1 equivalent.
5. Every language is finite state language.

Multiple Choices

1. A tree having n vertices will have -------------number of edges.

A) n B) n-1 C) n +1 D) Any

2. If K_n Complete graph is a tree ,then maximum value of n is -------

A) 1 B) 2 C) 5 D) Any

3. Which of the following tree is complete bipartite---------

A) $K_{1,2}$ B) $K_{2,3}$ C) $K_{3,4}$ D) $K_{1,2}, K_{2,3}, K_{3,4}$

4. For the finite state machine given by following fig-

The accepting State are

A) A B) A and E C) E D) A,B,C,D,E

Theory Questions (4-Marks)

1. Prove that a connected graph with n-vertices and n-1 edges is a tree.

2. Prove that a graph with n-vertices, n-1 edges that has no circuit is a tree.

3. Prove that there is unique path between every pair of vertices of a tree.

4. Prove that a graph in which there is unique path between any pair of vertices is a tree.

5. Draw all non-isomorphism trees on six vertices.

6. A tree has 2n pendent vertices, 3n vertices of degree 2 and n-vertices of degree 3 .Determine the number of vertices and edges in a tree.

7. A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many numbers of vertices of degree 1 does it have?

8. A tree has three vertices of degree 2, two vertices of degree 3, one vertex of degree 4. How many numbers of vertices of degree 1 does it have?

9. Prove that every tree is a bipartite graph but converse is not true.

10. Show that sum of degrees of all vertices of a tree with n-vertices is 2n-2.

11. A tree with n_2 vertices of degree 2 ,n_3 vertices of degree 3 and so on ,n_k vertices of degree k. How many leaves does it have?

12. Show that two states are in the same block π_k iff they are in the same block π_{k-1} and for any input letter their successors are in the same block in π_{k-1}.

13. Compute the partitions for the following finite state machine given by.

State		A	B	C	D	E	F	G	H
Input	0	B	A	G	H	A	H	A	A
	1	F	F	A	B	G	C	D	C
Output		0	0	0	0	0	1	1	1

14. Draw all possible non-isomorphic trees on 7-vertices.

15. Show that the language $L = \{0^i 1^j / i \geq j\}$ is not a finite state language.

16. Show that the language $L = \{0^i 1^j / i \leq j\}$ is not a finite state language.

17. Show that the language $L = \{0^k / k = 1^i, i \geq 1\}$ is not a finite state language.

18. Show that the language $L = \{1^i 0^j 1^{i+j} / i \geq 1, j \geq 1\}$ is not a finite state language

19. Show that the language $L = \{0^k / k = h^2, h > 0\}$ is not a finite state language

20. Draw all non-isomorphism trees on six vertices.

21. Define finite state machine and describe the machine graphically given by following table

State	Input			Output
	a	b	c	
S_0	S_1	S_2	S_5	0
S_1	S_2	S_3	S_6	0
S_2	S_3	S_4	S_6	0
S_3	S_4	S_5	S_6	0
S_4	S_5	S_6	S_6	0
S_5	S_6	S_6	S_6	0
S_6	S_1	S_2	S_5	1

22. For the finite state machine as shown in the table.

State		A	B	C	D	E	F	G
Input	0	B	B	A	B	F	A	B
	1	C	D	E	E	E	D	C
Output		0	0	0	0	0	1	1

Find all equivalent states and obtain an equivalent finite state machine with the small est number of states.

23. Write a note on equivalent states of finite state machines.

24. Compute the partition for the following finite state machine.

State	A	B	C	D	E	F	G	H
0	F	D	G	E	G	A	C	A
1	B	C	B	A	A	G	H	H
Output	0	0	0	1	0	1	1	1

25. Explain the graph with given specification exists or not.

a) A tree with all vertices of degree 2.

b) A tree with 10 vertices out of which 5 are even vertices.

c) A tree with 4 internal vertices and 6 terminal vertices.

d) A tree with 6 vertices having degree 1, 1, 1,1,3,3.

Theory Questions (6-Marks)

1. State and Prove Pumping Lemma for a finite state Languages in the literature.

2. Show that the Language $L = \{a^k b^k / k \geq 1\}$ is not a finite state language.

3. Show that the state language $L = \{a^k / k = i^2 \geq 1\}$ is not a finite state Language.3

4. Show that following two finite State machines are equivalent

State	Input		output
	0	1	
a	b	c	0
b	b	d	0
c	a	e	0
d	b	e	0
e	f	e	0
f	a	d	1
g	b	c	1

State	Input		output
	0	1	
a	h	c	0
b	g	b	0
c	d	b	0
d	d	c	0
e	h	b	0
f	d	e	1
g	h	c	1
h	a	e	0

5. Compute the partitions for the following finite state machine.

State	Input		output
	0	1	
A	B	F	0
B	A	F	0
C	G	A	0
D	H	B	0
E	A	A	0
F	H	C	1
G	A	D	1
H	A	C	1

6. Prove that a tree with two or more vertices has at least two pendant vertices.

7. Prove that a tree with n-vertices must have n-1 edges.

8. Show that there exist a tree whose vertices have degree $d_1, d_2, d_3 - - - - - d_n$ if for $n \geq 2$;

Let $d_1, d_2, d_3 - - - - - d_n$ be n positive integers such that $\sum_{i=1}^{n} d_i = 2n - 2$.

9. Let T be a tree with 50 edges. The removal of a certain edges from T yields two disjoint tree T_1 and T_2 .Given that number of vertices in T_1 is equal to number of vertices in T_2.Find the number of vertices and the number of edges in T_1 and T_2

10. Prove that a complete bipartite graph $K_{m,n}$ is a tree iff either m = 1 or n = 1.

11. Show that the language $L = \{a^k b^k / k \geq 1\}$ is not a finite state language.

12. Show that the language L = {xx/ x is a string of $0^{'s}$ & $1^{'s}$ }. Is not a finite state language.

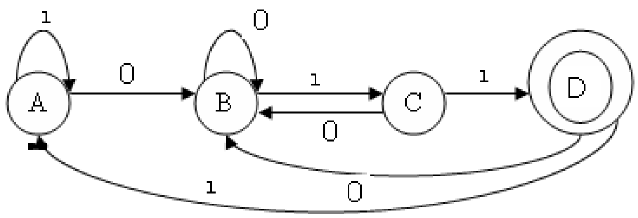
13. For the finite state machine ,Show that in the table

State	A	B	C	D	E	F	G	H
0	F	D	G	E	G	A	C	A
1	B	C	B	A	A	G	H	H
Output	0	0	0	1	0	1	1	1

a) List all 0-equivalent state.

b) Find all equivalent states and obtain an equivalent finite state machine with the smallest number of states.

14. Define Finite state language and obtain finite state language of the machine given by following digraph also obtain its state transition table.



15. Draw all trees with a) 5 vertices

b) 8 vertices

Marks – 4

1. A finite state Machine M = {S, I, O, f, g} is given by state transition table

State	Input			Output
	a	b	c	
S_0	S_1	S_2	S_5	0
S_1	S_2	S_3	S_6	0
S_2	S_3	S_4	S_6	0
S_3	S_4	S_5	S_6	0
S_4	S_5	S_6	S_6	0
S_5	S_6	S_6	S_6	0
S_6	S_1	S_2	S_5	1

Describe a finite state machine graphically.

2. State a prove pumping Lemma for finite state languages in the literature.

3. Show that the language L = $\{a^k b^k / k \geq 1\}$ is not finite state machine.

4. Show that the language L = $\{a^k / k = i^2 \geq 1\}$ is not finite state machine.

5. For the finite state machine, Shown in the table,

State	Input		output
	0	1	
a	f	b	0
b	d	c	0
c	g	b	0
d	e	a	1
e	g	a	0
f	a	g	1
g	c	h	1
h	a	h	1

List all 0-equivalent states .Find all equivalent state and obtain an equivalent finite Machine with the smallest number of states

Mathematics Paper-III (B)

Unit –IV

Relations and Lattices

2 marks

1. Define Partially ordered relation

2. Define maximal and minimal element in a poset

3. Define comparable element in a poset

4. Define upper bound and lower bound in a poset

5. Define Least upper bound and greatest lower bound in a poset

6. Define chain and ant chain in a poset

7. What is mean by discrete numeric function

8. If a is numeric function and t be positive intger, then how we write $S^{i}a \ and \ \ S^{-i}a$.

9. Define forward difference Δa and backward difference ∇a of numeric function a.

10. Define convolution of two numeric function a and b.

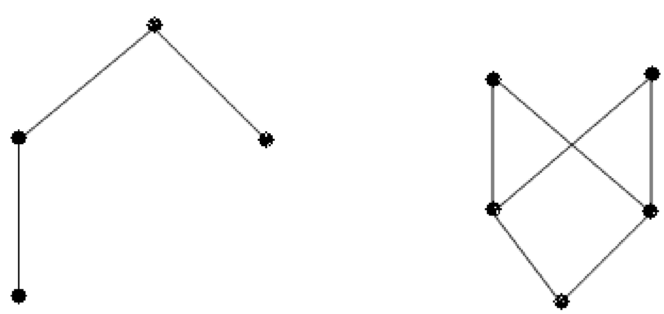
11. Define asymptotic dominance.

12. Define generating function of numeric function.

13. If f(z) is generating function of numeric function a, then write generating function of $S^{i}a \ and \ \ S^{-i}a$.

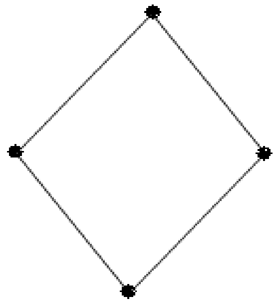
14. $A=\{a,b,c,d\}$, $R=\{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c),(c,b),(d,d),(d,b)\}$,Is (A,R) poset ? Explain.

15. For the poset described in following Hasse diagrams.



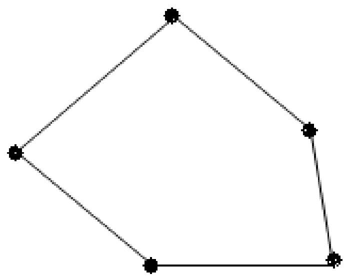
Find maximal and minimal element

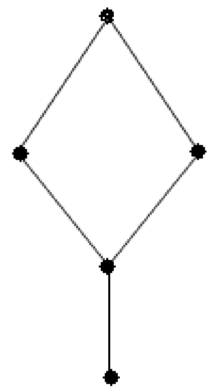
16. Consider the poset described by following Hasse diagram. Find Comparable elements and Non-comparable elements.

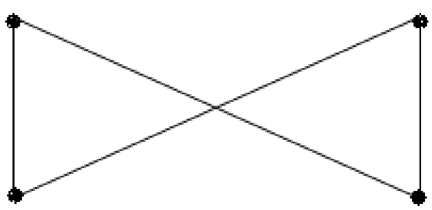


17. .Define Lattice.

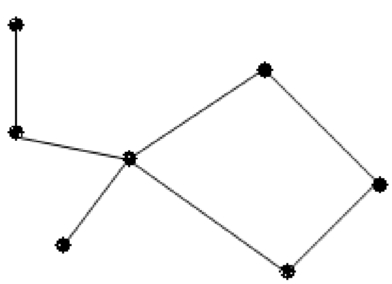
18. . Is the poset Lattice? Explain.







19. Find maximal and minimal element of partially ordered set described in following figure.



20. In a set of 12 integers there are two integers, whose difference is divisible by 11.State true or false.

21. Amongst any eight people there are at least two who were born in the same day of week. State true or false.

22. If $\begin{aligned} ar &= 3r \quad , 0 \le r \le 5 \\ &= r+1, \ r \ge 6 \end{aligned}$ then a = {--------------}.

23. If a is numeric function with $ar = 3r^3 - 1, r \ge 0$ and $\alpha = 5$ then , $\alpha a = ---$

24. If a and b are numeric functions such that $\begin{aligned} ar &= 5^2 \quad ,if \ r \ge 0 \\ br &= 7^2 \quad ,if \ r \ge 0 \end{aligned}$ then $C_r = ----$

Where $C = a \times b$

25. Let $\begin{aligned} a_r &= 1 + 0.2r \qquad if \ r \ge 0 \\ b_r &= (1.03)^{4r} \qquad if \ r \ge 0 \end{aligned}$ then ----------asymptotically dominates------

26. If $a = \{4^0, 4^1, 4^2, 4^3 -----\}$ then A(z) = ----------

27. If a is numeric function with $a_r = 3^r, r \ge 0$ then A (z) = ----------.

28. If $a_r = 3^r, b_r = 2^r \quad r \ge 0$ and $C_r = a_r + b_r$ then generating function C(z) = -----

29. Given $a_r = 5, r \ge 0$ then numeric function a = { -----} and generating function A(z) =----

30. The generating function $A(z) = \dfrac{1}{1+z}$ then compounding numeric function a =------

31. Determine the generating function for the numeric function a = { 0 ,1,2,3,-----}

32. If $a_r = \dfrac{1}{r!}, \forall r \ge 0$ then A(z) = ----------

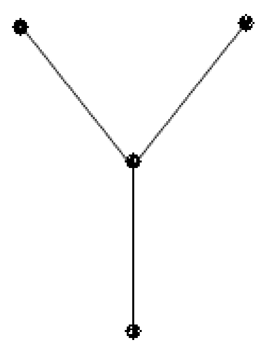
33. If $a_r = 1, \forall r \ge 0$ then A(z) = ---------

34. Is (z,>) is poset? Explain.

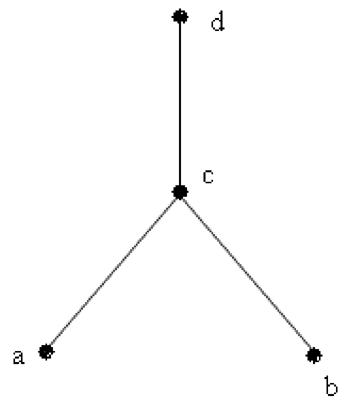
35. Which of the following pair of elements is comparable in the poset $(z^+, /)$?

A) 2, 4 B) 4, 6 C) 2, 5 D) 10, 12

36. Is the following poset a lattice?



37. For the poset



The minimal element is

A) a B) b C) a and b D) c

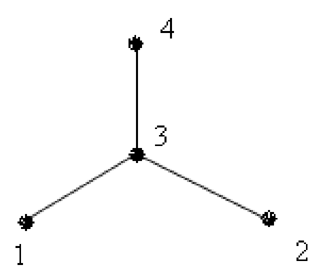
38. If $ar = (-1)^{r+1}, |r| < 1$ then the generating function A(z) is --------

A) $\frac{1}{1-z}$ B) e^z C) 2^z D) $\log(1+z)$

39. A= {a,b,c} , $(P(z), \leq)$ is poset .The maximal element is --------

A) {a} B) ϕ C) {a, b} D) {a, b, c}

40. Describe the order pairs in the relation determined by the Hasse diagram on thee set A = {1, 2, 3, 4}



04 or 06 marks

1. Define POSET. If A = {a,b,c,d} and

R = {(a, a),(a,b),(a,c),(a,d),(b,b),(c,c),(c,b),(d,d),(d,b)}.Is (A,R) a poset ? Justify.

2. Let N = {1, 2, 3, 4------} R is relation defined by, $xRy \text{ iff } x \le y$ .Show that (N, R) is Poset.

3. A be any set. P (A) is power set of A.Let R be relation

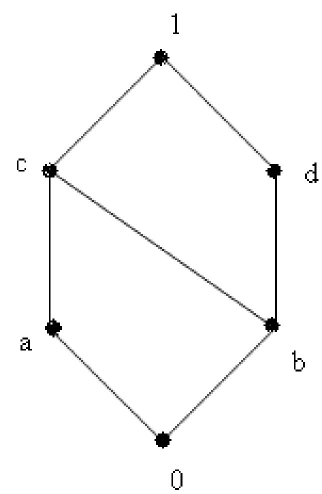
$R = \{(B,C)/B,C \in P(A), B \subseteq C\}$, Show that (A, R) is poset.

4. Explain what is mean by Hasse diagram of poset.

A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be poset. The partially ordering relation is xRy iff "x divides y " $x, y \in A$ .Draw the Hasse diagram of the poset A.

5. Let A = { a, b,c }, P(A) is power set of A. $(P(A), \subseteq)$ is poset.Draw the Hasse diagram Of the poset.Find Maximal and minimal elements.

6. Define least upper bound and greatest lower bound in a poset. For the poset with Hasse diagram.



a) Find l.u.b. and g.l.b. of a & b

b) Find l.u.b. and g.l.b. of a & d

7. Let $D_{30} = \{1,2,3,5,6,10,15,30\}$ set of all positive divisors of 30. The relation "divides" be partially ordered relation on D_{30} .

a) Draw Hasse diagram.

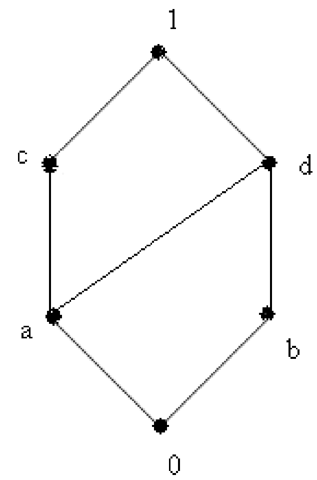
b) Find all lower bounds of 10 and 15.

c) Find g.l.b. of 10 and 15.

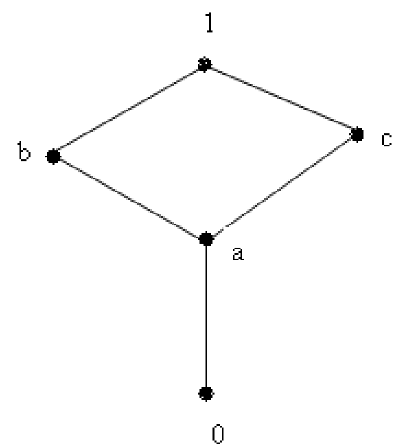
d) Find all upper bounds of 10 and 15.

e) Find l.u.b. of 10 and 15.

8. Define lattice. Show that the poset in following figure is lattice.



9. Show that the poset in following figure is lattice.

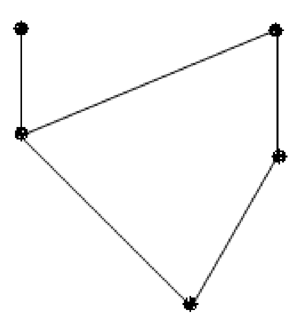


10. Let $(A, \leq)$ be poset consisting of mn + 1 elements .Prove that either there is an antichain consisting of m +1 element or there is a chain of length n +1 in A.

11. Draw the Hasse diagram of the following posets under partially ordered relation "divides" Indicate which are chains, a) A = { 2, 4, 12, 24 }

b) B = {1, 3, 5, 15, 30}

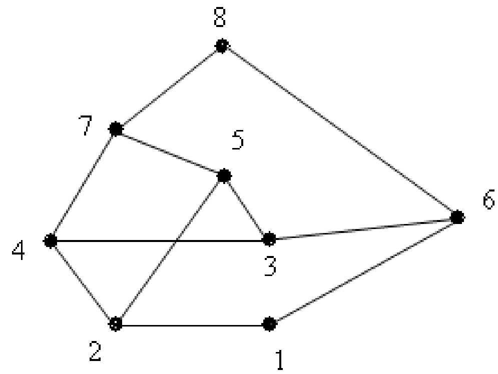
12. Find the ant chains with the greatest number of elements in the poset with following Hasse diagram.



13. In the poset P shown in the following figure, Find upper bound and l.u.b. for

a) A = {2, 3}

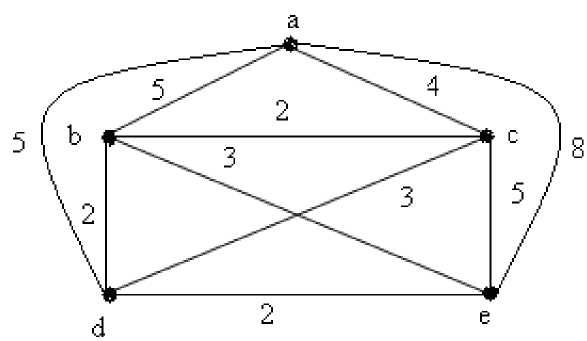
b) B = {4, 6}



14. State generalized pigeonhold Principle. Find the minimum number of students in a class to be sure that three of them born in some month.

15. Suppose a laundary bag contains many red, white and blue socks. Find the minimum number of socks that one pairs of same colour.

16. Find the time complexity of the nearest neighbour algorithm for traveling Salesman problem of following graph.



17. Given $a_r = 1, \quad if \ \ 0 \le r \le 2$ $\quad = 3r \ \ if \ \ r \ge 3$ and $b_r = 2^r + 1, \quad if \ \ 0 \le r \le 1$ $\quad = r - 5 \quad if \ \ r \ge 2$

Find a) $a_r + b_r$ b) $a_r . b_r$

18. Let a be numeric function where

$$\begin{aligned} a_r &= 1, \quad if \ \ 0 \le r \le 10 \\ &= 2 \ \ if \ \ r \ge 11 \end{aligned}$$

Find $S^5 a$ and $S^{-7} a$.

19. Let a be numeric function where

$$\begin{aligned} a_r &= 1, \quad if \ \ 0 \le r \le 2 \\ &= 3^r \ \ if \ \ r \ge 3 \end{aligned}$$

Find Δa and ∇a .

20. If a and b are numeric functions such that

$$a_r = 0, \quad if\ 0 \le r \le 2 \qquad \text{and} \qquad b_r = 3 - 2^r, \quad if\ 0 \le r \le 1$$
$$= 2^{-r} + 5 \quad if\ r \ge 3 \qquad\qquad = r + 2, \quad if\ r \ge 2$$

Find a) c = a + b such that $C_r = a_r + b_r$

b) d= a.b such that $d_r = a_r.b_r$

21. If $a_r = 0, \quad if\ 0 \le r \le 2$ and $b_r = (3 + r) \quad if\ 0 \le r \le 1$

$= 5^r \quad if\ r \ge 3 \qquad = 2^r \quad if\ r \ge 2$

Find a) $C_r = a_r + b_r$ b) $d_r = a_r.b_r$

22. Determine $a \times b$ if

$$a_r = 1, \quad if\ 0 \le r \le 2 \qquad \text{and} \qquad a_r = r + 1 \quad if\ 0 \le r \le 2$$
$$= 0 \quad if\ r \ge 3 \qquad\qquad = 0 \quad if\ r \ge 3$$

23. The numeric function a is defined as

$$a_r = 2 \quad if\ 0 \le r \le 3$$
$$= 2^{-r} + 5 \quad if\ r \ge 4$$

Determine a) $S^2 a$ b) $S^{-2} a$.

24. The numeric function a is defined as

$$a_r = 2 \quad if\ 0 \le r \le 3$$
$$= 2^{-r} + 5 \quad if\ r \ge 4$$

Determine Δa and ∇a .

25. If $a_r = 3^r, b_r = 5^r, r \ge 0$ and $c = a \times b$ .Determine the generating function C(z).

26. Determine the generating function of numeric function

a) $a_r = 3^r + 4^{r+1} \quad if\ \ r \ge 0$

b) $a_r = 5 \quad if\ r \ge 0$

27. Determine the discrete numeric function for the following generating function

$$A(z) = \frac{3 - 5z}{1 - 2z - 3z^2}$$

28. Determine the numeric functions c and d if a) $C_r = a_r + b_r$ b) $d_r = a_r.b_r$

Given $a_r = 0 \quad if\ 0 \le r \le 2$ and $a_r = 3 - 2^r \quad if\ 0 \le r \le 1$

$= 2^{-r} + 5 \quad if\ r \ge 3 \qquad = r + 2 \quad if\ r \ge 2$

29. Define the convolution of two numeric functions a and b .Find the convolution of the numeric functions a and b where $a_r = 9^r$ and $b_r = 3^r$.

30. Find the generating function of numeric function whose rth term.

$$a_r = 0 \quad if\ r\ is\ odd$$
$$= 2^{r+1} \ if\ r\ is\ \text{even}$$

31. Find the numeric function corresponding to the generating function $\frac{2+3z-6z^2}{(1-z^2)}$

32. Find the numeric function corresponding to the generating function $\frac{z^2}{(1-2z)}$

33. Find the numeric function corresponding to the generating function $a=\left(0,3\times1\times2^1,3\times2\times2^2,3\times3\times2^2,3\times4\times2^2 ------\right)$

34. Find the discrete numeric function for the generating function $A(z)=\frac{3+7z}{(1+4z)(1-z)}$

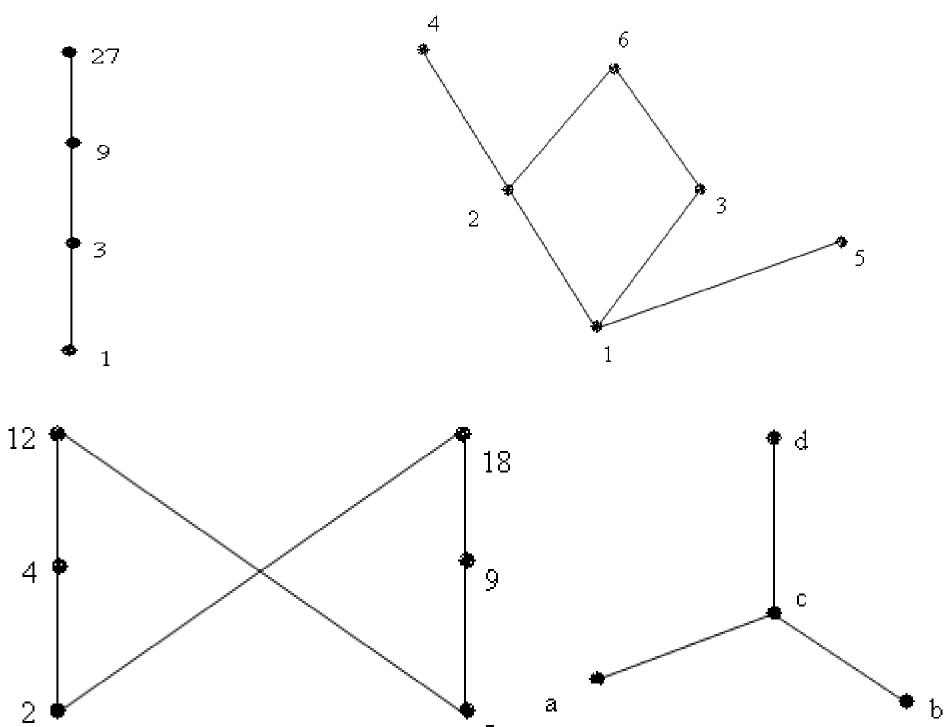
35. Find the discrete numeric function for the generating function $A(z)=\frac{z^3}{1+3z}$

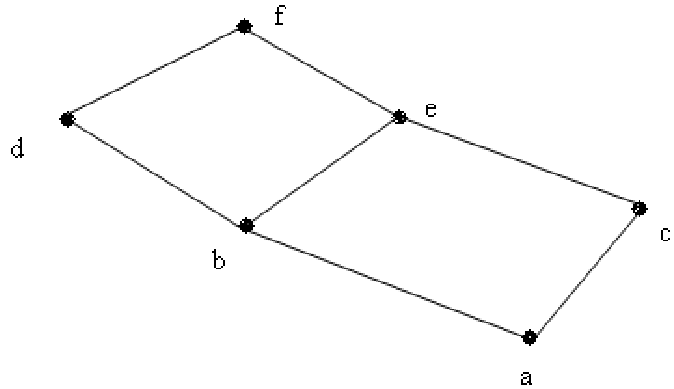
36. Find the discrete numeric function for the generating function $A(z)=\frac{1}{(5-6z+z^2)}$

37. Find a closed form of the generating function for the numeric function $a_r=\binom{6}{r}$ for all r = 0,1,2-------

38. Find a closed form of the generating function for the numeric function $a_r=5r+2$ for all $r\geq 0$

39. Determine the maximal and minimal elements for the following posets





40. Which of the following are partial orders?

A) The relation $R = \{(a,b) \in Z \times Z : |a-b| \leq 1\}$ on Z.

B) The relation $R = \{(a,b) \in Z \times Z : |a| \leq |b| \leq 1\}$ on Z

C) The relation $R = \{(a,b) \in Z \times Z : a \text{ divides } b \text{ in } z\}$ on Z

41. Which of the following pairs of elements are comparable in poset $(z^{+}, /)$.

A) 2, 4 B) 4, 6 C) 5, 5

42. Let A = { 1,2,3,4 } and consider the relation R = { (1,1) ,(1,2),(1,3), (2,2) (3,2),(3,3) (4,2),(4,3),(4,4) } Show that R is partial ordering and draw its Hasse Diagram.

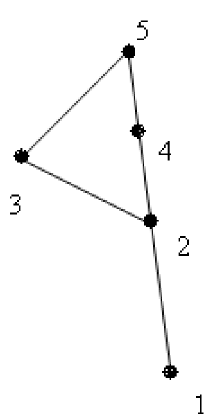
43. Draw the Hasse diagram for the divisibility relation on each of the following sets.

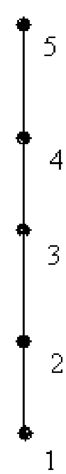
a) A = { 2,3,4,12,24,36 } b) A = {3,6,12,24,48 }

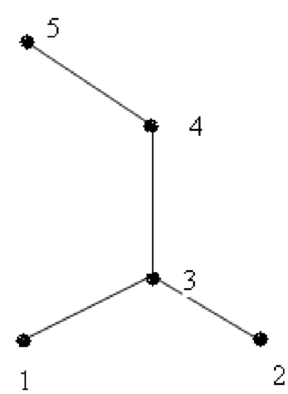
44. Let D_m denote the positive divisor of m ordered by divisibility. Draw the Hasse diagram of

$a)D_{12}$ $b)D_{15}$ $c)D_{16}$ $d)D_{17}$

45. Let A = {1, 2, 3, 4, 5}.Determine the relation represented by the following Hasse diagram.

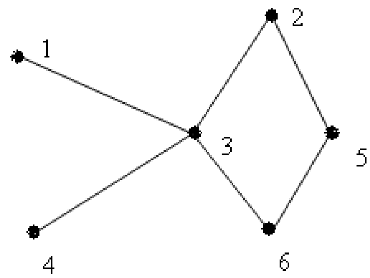






46. The set P ({a, b, c}) is partially ordered with respect to the subject relation $\subseteq$.Find a chain of length 3 in P ({a, b,c}).

47. Let A = {1, 2,3,4,5 6} be ordered set as shown in fig.



Find a) all minimal and maximal elements.

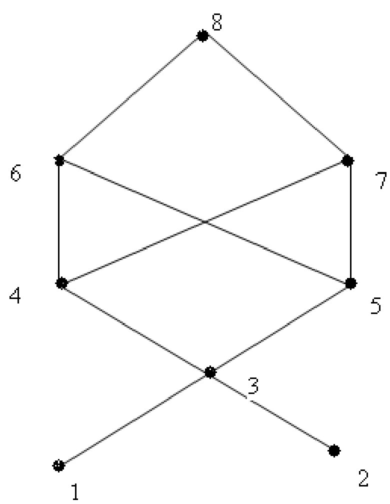
b) Upper bounds of 3, 5.

c) Lower bound of 3, 5.

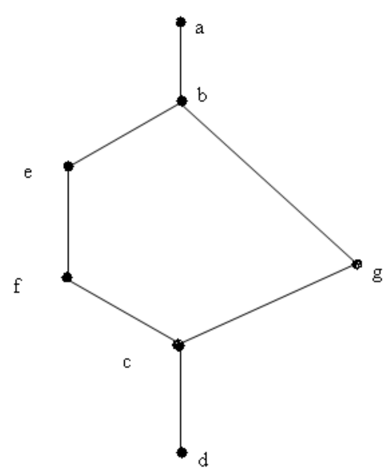
48. Consider the poset A = { 1,2,3,4,5,6,7,8 } under the partial order whose Hasse diagram is as shown below. Consider the set B = {1,2 } and C = { 3,4,5 } of A. Find

a) All the lower and upper bounds of B and C.

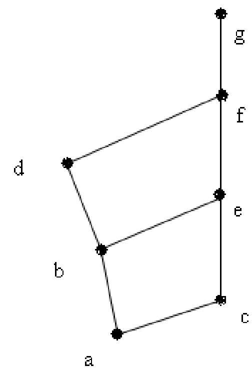
b) glb(B) , lub(B), glb(C), and lub(c)



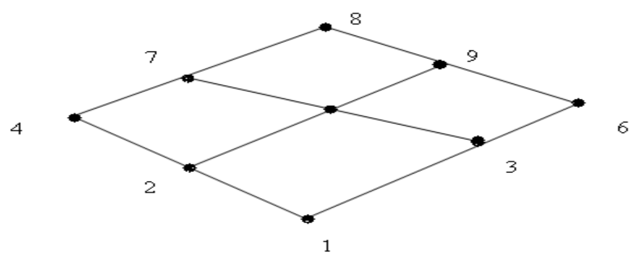
49. Is the following poset lattice? Justify



50. Is the following poset a lattice? Justify.



51. Is the following poset a lattice? Justify



52. Ramesh deposits Rs. 200 in a saving account at an interest rate of 9% per year. Compound annually, If a_r denote the amount in the account after r year, determine the numeric function a.

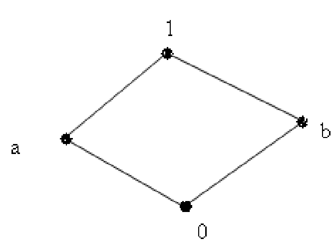
53. In a process control system, a monitoring device measures the temperature inside a chemical reacting chamber once every 30 sec. Let a_r denote the rth reading in degrees centigrade. Determine an expression for a_r, if it is known that the temperature rises from $100^0 C$ to $120^0 C$ at constant rate in the first 300 sec. and stays at $120^0 C$ from then on.

54. Find the minimum number of element that one needs to take from the set S = { 1,2,3,4,5,6,7,8,9 } to be sure that two of the numbers add upto 10.

55. Find the maximal and minimal elements of the set A = {2, 3, 5, 7, 11-----} of prime numbers ordered by divisibility.

56. List the elements of D_{1001} (all positive divisors of 1001.) Draw the Hasse diagram for D_{1001} with partial ordering relation "divides"

57. Define covering relation in a poset Consider the poset $(A,\leq)$ described in figure



a) Indicate the poset in covering relations.
b) Write the relation in ordered pairs.

6 marks

1. Let $(A, \leq)$ be partially ordered set. Suppose the length of the longest chain in A is n. Prove that elements in A can be partitioned into n disjoint antichains.
2. Suppose n distinct numbers are stored in n resisters $x_1, x_2, -----x_n$ .Design an algorithm to determine the largest of these n numbers. By algorithm LARGEST1 .Also find time complexity.
3. Suppose n distinct numbers are stored in n resisters $x_1, x_2, -----x_n$ .Design an algorithm to determine the largest of these n numbers. By algorithm LARGEST2 .Also find time complexity.
4. By sorting the n numbers stored in resisters $x_1, x_2, -----x_n$,Rearrange them such that the rearranged contents of registers $x_1, x_2, -----x_n$ are in ascending order by BUBBLE SORT algorithm.
5. Prove that relation, ${}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$ by using generating function.

Mathematics Paper-III (B)

Unit –V

Boolean Algebra

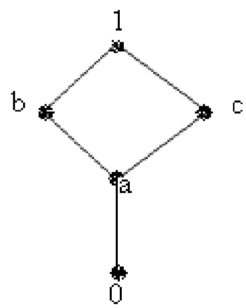
2 marks

1. Define Universal upper bound and Universal lower bound.

2. The dual of the statement

$(a \vee b) \wedge c = (b \wedge c) \vee (c \wedge a)$ is --------

3. Write the dual of lattice depicted in following figure



4. State a) Idempotent property for join and meet.

b) Absorption property for join and meet.

5. Define distributive lattice.

6. Define complement of an element in lattice

7. Define complement lattice.

8. Define sub-lattice of lattice (L, <).

9. Define Modular lattice

10. State De-Morgans laws in Boolean algebra.

11. What is mean by an atom in Boolean algebra?

12. If $E(x_1, x_2, x_3) = \left[(x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})\right] \wedge (\overline{x_2} \vee x_3)$ is Boolean expression over the Boolean algebra $\left(\{0,1\}, \vee, \wedge, -\right)$ Then E(0,1,0) = --------

13. Let $E(x_1, x_2, x_3) = x_1 \vee x_2 \vee \overline{x_3}$ be Boolean expression over the Boolean algebra $\left(\{0,1\}, \vee, \wedge, -\right)$ then $E(x_1, x_2, x_3) = ------$ for

$a) x_1 = 1, x_2 = 0, x_3 = 1$

$b) x_1 = 0, x_2 = 0, x_3 = 1$

$c) x_1 = 0, x_2 = 1, x_3 = 1$

14. Explain Min term Boolean expression and give example.

15. Explain Max term Boolean expression and give example.

16. The number of different Boolean function of degree n is ---------.

17. The Cartesian product of two lattices is always a lattice, State true or false.

18. The dual of the statement

$(a \wedge b) \vee c = (b \vee c) \wedge (c \vee a)$ is --------

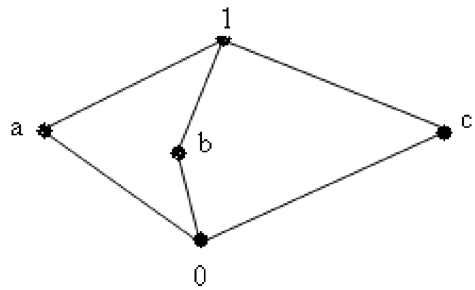
19. Every distributive lattice is modular. State true or false.

20. In a complement lattice if $b \wedge \bar{c} = 0$ then Show that $b \leq c$.

21. Define bounded lattice.

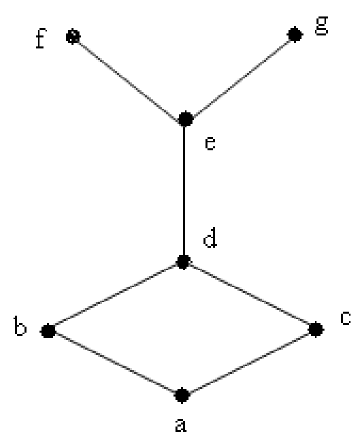
22. Let $(L, \leq)$ be a bounded lattice with lower bound 0 and upper bound 1 .Show that 0 and 1 are complements of each other.

23. The following lattice is distributive .State true or false.



24. Every chain is lattice. State true or false.

25. The following poset is lattice. State true or false.



26. Choose correct answers. If L is lattice ,then for every a and b in L. $a \vee b = b$ iff

A) $a \geq b$ B) $a \leq b$ C) a >b D) None of these

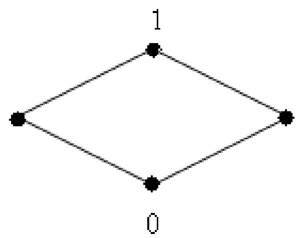
27. Choose correct answers. If L is lattice ,then for every a and b in L. $a \wedge b = a$ iff

A) $a \geq b$ B) $a \leq b$ C) a >b D) None of these

28. Choose correct answers. The property $a \vee (a \wedge b) = a, a \wedge (a \vee b) = a,$ in lattice is called ---

A) Commutative B) Idempotent C) Absorption D) Associative

29. Choose correct answers. In the following lattice the complement of a_1 is



A) a_2 B) 1 C) 0 D) a_1

30. Choose correct answers. The property $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for a , b, c in lattice is called --------

A) Commutative B) Distributive C) Absorption D) Associative

31. In a distributive complemented lattice every element has unique complement .State true or false.

32. In a complement lattice $(L, \leq)$ Show that $a \leq b \; iff \; a' \vee b = 1$

33. In a complement lattice $(L, \leq)$ Show that $a \leq b \; iff \; a \wedge b' = 1$

34. Let $B = (\{0,1\}, +, ., /)$ are defined as

+	1	0
1	0	1
0	1	0

•	1	0
1	1	0
0	0	0

X = 111011 , Y = 101011 Find X+Y and X.Y

35. Give examples of non-distributive lattice .

Boolean algebra

1. Show that in any lattice $(L,\leq)$, $a \vee b \leq a \leq a \vee b$ and $a \vee b \leq b \leq a \vee b$

2. Let $(L,\leq)$ be a lattice with universal bounds 0 and 1 then show that for every $a \in L$

a) $a \vee 1 = 1$, $a \wedge 1 = a$

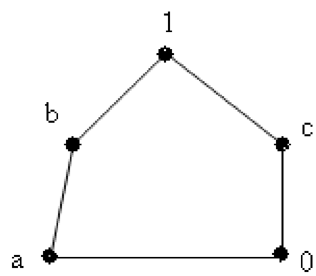
b) $a \vee 0 = a$, $a \wedge 0 = 0$

3. Explain " Principle of dually"

4. For any a, b, c, d in $(L,\leq)$ Prove that if $a \leq b$ and $c \leq d$ then a) $a \vee c \leq b \vee d$

b) $a \wedge c \leq b \wedge d$

5. Consider the Hasse diagram $(n_5,\leq)$ .Is distributive lattice ?Justify



6. Let $(L,\wedge,\vee)$ be an algebra system .Any a,b,c be element in lattice $(L,\leq)$ .Prove that

a) $a \vee a = a$, $a \wedge a = a$

b) $a \wedge (a \vee b) = a$, $a \vee (a \wedge b) = a$

7. Let $(L,\wedge,\vee)$ be an algebra system .Any a,b,c be element in lattice $(L,\leq)$ .Prove that

a) $a \wedge b = b \wedge a, a \vee b = b \vee a$

b) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$, $(a \vee b) \vee c = a \vee (b \vee c)$

8. Let $(L,\leq)$ be lattice and $a,b,c \in L$ a) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

b) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Are above condition are equivalent to each other.

9. Show that a) $a \leq b \Leftrightarrow a' \vee b = 1$

b) $a \leq b \Leftrightarrow b' \leq a'$ Where $(L,\leq)$ is complement lattice

10. Show that ,Let $(L,\leq)$ is complement lattice ,

a) $(a \wedge b)' = a' \vee b'$ b) $(a \vee b)' = a' \wedge b'$

11. Prove that If $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ for some a, then b = c Where $(L,\leq)$ is distributive lattice.

12. Prove that in a complement lattice $(L,\leq)$,

a) $a \leq b$ *iff* $a \wedge b' = 0$ b) $a \leq b$ *iff* $b' \leq a'$

13. Let $(L, \wedge, \vee)$ be algebraic system defined a lattice $(L, \leq)$, Prove that

a) $a \wedge (a \vee b) = a \,\&\, a \vee (a \wedge b) = a$

b) $(a \wedge b) \wedge c = a \wedge (b \wedge c) \,\&\, (a \vee b) \vee c = a \vee (b \vee c)$ Where a, b, c in L

14. Show that every sub lattice of distributive lattice is distributive lattice.

15. Let $D_{12} = \{1, 2, 3, 4, 6, 12\}$ the divisors of 12.Draw Hasse diagram.

16. Show that every distributive lattice is modular.

17. Let a,b any two element in $(L, \leq)$, Prove that $a \wedge b = b \Leftrightarrow a \vee b = a$

18. Let $D_{15} and\ D_{16}$ are positive divisor of 15 and 16 respectively. Draw Hasse diagram.

19. Explain Boolean lattice and Boolean algebra.

20. Show that a and b are element in Boolean algebra.

a) $\overline{a \vee b} = \overline{a} \wedge \overline{b}$

b) $\overline{a \wedge b} = \overline{a} \vee \overline{b}$

21. Draw Hasse diagram $(D_{30}, /)$.

22. Find the atoms of Boolean algebra.

a) B^2 b) B^3

23. Define $f : B^3 \rightarrow B$ in truth table

X_1	X_2	X_3	$f(x_1, x_2, x_3)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Find Boolean expression in CNF.

24. In Boolean algebra $(B, \wedge, \vee, -)$ express the Boolean function

$f(x_1, x_2, x_3) = [\overline{(x_1 \wedge \overline{x_2}) \vee (x_1 \wedge x_3)}] \vee \overline{x_3}$ Find Boolean expression in CNF

25. Construct the truth table for the Boolean expression $E(x_{1,} x_2, x_3) = x_1 \vee x_2 \vee \overline{x_3}$

26. Prove Hasse diagram $D_{70} and\ D_{36}$ with partial order relation "divides"

27. Show that a,b,c in lattice $(L,\leq)$,If $a \leq b$ then $a \vee (b \wedge c) \leq b \wedge (a \vee c)$

28. Let a,b in lattice $(L,\leq)$, Show that $a \wedge b = b \Leftrightarrow a \vee b = a$

29. Let L = $D_{18} = \{1,2,3,6,9,18\}$ the divisor of 18 ordered by divisibility. Find

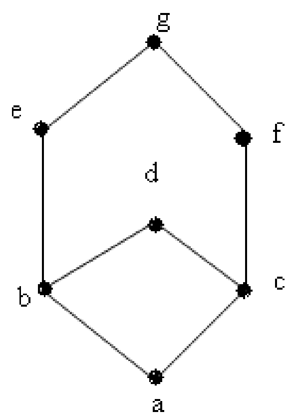
a) The lower bound and upper bound of L

b) The complement of 6

c) Is L a complement lattice?

30. Consider Hasse diagram the lattice $(L,\leq)$, Determine weather M,N,O and P is a sub lattice of L

Where $M = \{a,b,c,g\}$ $N = \{a,b,f,g\}$ $O = \{b,d,e,g\}$ $P = \{a,d,e,g\}$



31. Let M is a sub lattice of distributive lattice L Show that M is sub lattice.

32. Prove that every distributive lattice is modular lattice. Is the Converse true?

33. Let $B = D_{70} = \{1,2,5,7,10,14,35,70\}$

Define + on B by $a + b = lcm(a,b)$,

- On B by $a \bullet b = \gcd(a,b)$, $\forall a,b \in D_{70}$

' On B by $a' = \dfrac{70}{a}$, $\forall a,b \in D_{70}$

Show that $(D_{70}, +, \bullet, ')$ is a Boolean algebra

34. Let $E(x_1,x_2,x_3) = x_1 \vee x_2 \vee \overline{x_3}$ be Boolean expression over the Boolean algebra $(\{0,1\},\vee,\wedge,-)$ Evaluate $E(x_1,x_2,x_3)$ for the following assignment values -

$a) x_1 = 1, x_2 = 0, x_3 = 1$

$b) x_1 = 0, x_2 = 0, x_3 = 1$

$c) x_1 = 0, x_2 = 1, x_3 = 1$

35. Express the function is the truth table below in

a) Disjunctive normal form

b) Conjunctive normal form

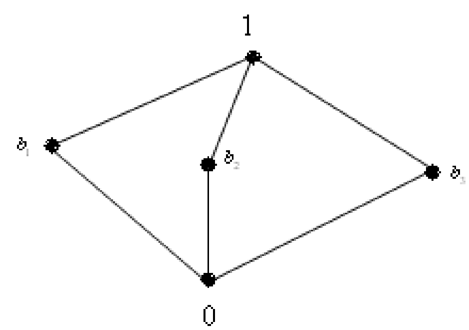
X_1	X_2	X_3	$f(x_1, x_2, x_3)$
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

36. Let L be a lattice and let a and b be elements of L such that $a \le b$. The interval [a, b] is defined as the set of all $x \in L$ such that $a \le x \le b$. Prove that [a, b] is a sub-lattice of L.

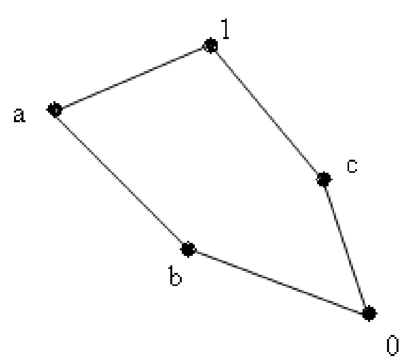
37. Show that an example that the union of two sub-lattices may not be a sub-lattice.

38. Show that the lattice $(L^3, \le_3)$ of 3 tuples of 0 and 1 is Complement

39. Show that the lattice given in the following diagram is not distributive.



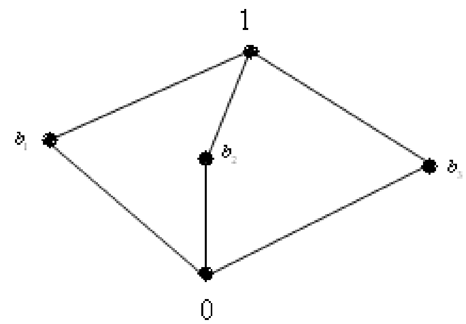
40. Show that the lattice given in the following diagram is not distributive.



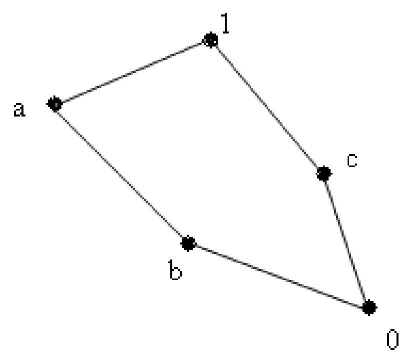
Pentagoan

41. Show that in a distributive lattice, If an element has a complement then this complement is unique.

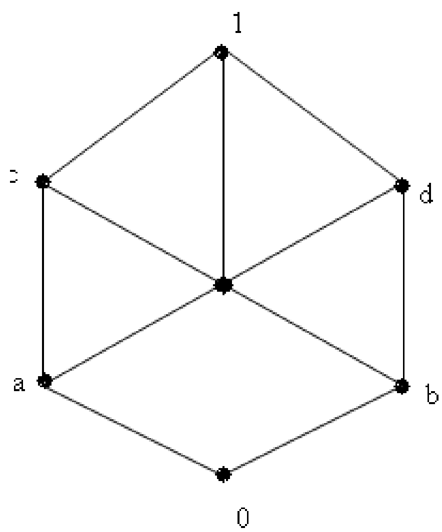
42. Prove that the lattice given by the following diagram is modular.



43. Show that the pentagonal lattice is not modular.



44. Consider the lattice L in figure



45. Which of the following is sub-lattice of L?

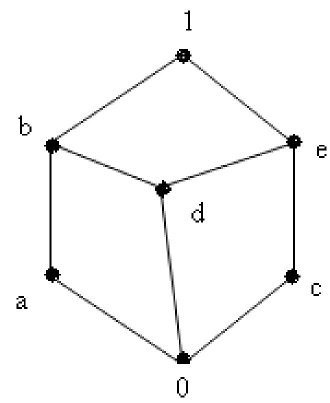
$L_1 = \{0, a_1, a_2, 1\}$ $L_2 = \{0, a_1, a_5, 1\}$ $L_3 = \{a_1, a_3, a_4, 1\}$

46. Is L distributive? Justify

47. Is L a complement lattice? Justify

Consider the lattice L in following figure

48. Find all sub-lattices with five elements



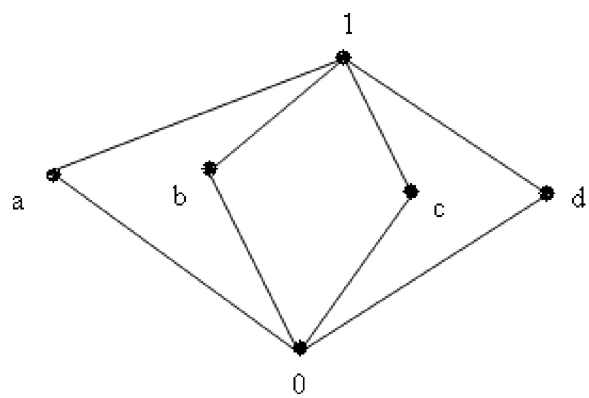
49. Find Complements of a and b if exists

50. Is L distributive? Justify

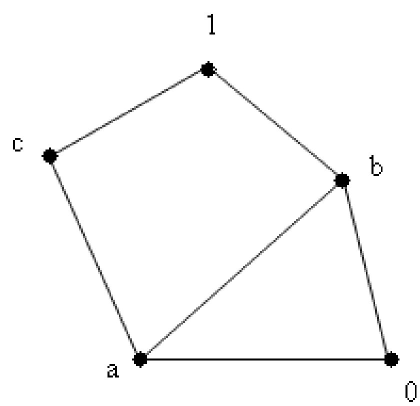
51. Is L a complement lattice? Justify

52. Find all sub-lattices of D_{24} that contain five or more elements

53. Show that lattice L represented by thee diagram is complement ,modular but not distributive



54. Show that lattice L represented by thee diagram is modular, distributive but not complemented.



55. Find complement of each element of D_{42} .

56. Find the number of different Boolean function of degree n.

57. In Boolean algebra $(B,+,\bullet./)$ express the Boolean function

$f(x,y,z)=(x\cdot y'+x\cdot z)'+z'$ in conjunctive normal form.

58. Construct the truth table for the Boolean expression $E(x_1,x_2,x_3)=x_1\vee x_2\vee\overline{x_3}$

59. Verity whether the following Boolean expression E_1 and E_2 are equal or not.

$E_1=(x_1\wedge\overline{x}_1)\vee(x_2\wedge\overline{x}_2)\vee(\overline{x_3\vee x_2})$

$E_2=\overline{x_1}\wedge\overline{x_2}$

60. In the Boolean algebra $(B,\wedge,\vee,-)$ express the Boolean function

$f(x_1,x_2,x_3)=\overline{x_1}\vee(x_2\wedge x_3)$ in its CNF