

***NORTH MAHARASHTRA UNIVERSITY
JALGAON***

QUESTION BANK

FOR

F.Y.B.Sc

STATISTICS

PAPER-II

**PROBABILITY & PROBABILITY
DISTRIBUTIONS**

(With effect from June 2007)

Paper – II

Probability & Probability Distributions

Chapter-1 Sample Space & Events

A) Questions for 2 marks

1. Give one example each of deterministic & Non – deterministic experiments.
2. Explain random experiment.
3. Define sample space & give two examples of it
4. Draw Venn diagram of $A \cup B$; where A & B are any two events of Ω
5. Draw Venn diagram of $A \subset B$ and $B \subset A$ where A & B are any two events of Ω
6. Define an elementary event & a compound event
7. Give two examples of countably infinite sample space.
8. Give one example of sure event & impossible event.
9. Explain mutually exclusive events.
10. Explain mutually exhaustive events.
11. Give example of events, which are mutually exclusive & exhaustive.
12. Let $\Omega = \{1,2, \dots, 10\}$, A is event of even numbers & B is event of the numbers divisible by 3. What will be $A \cup B$ & $A \cap B$?
13. When an event is said to be occurred? Explain
14. What do you understand by equally likely events?
15. Consider an experiment of rolling of two dice. Write the event that at least one number 2 will occur on uppermost face.
16. A coin is tossed until first head is obtained. Write down the sample space of this expt.
17. Explain concept of complement of an event by Venn- diagram.
18. Let A, B & C be any three events defined on sample space Ω .
19. Explain by Venn – diagram, simultaneous occurrence of A & B.
20. Explain discrete sample space.

B) Questions for 4 Marks

1. What do you mean by an experiment? What are different types of experiments? Explain.
2. Define with example each of the following
 - i) Sample space
 - ii) Discrete sample space
 - iii) Event
 - iv) Simple event

3. Write a note on sample space.
4. Explain the concept of mutually exclusive & exhaustive events
5. Explain the concept of relative complementation by way of Venn- diagram & also with a suitable example.
6. Give four examples each of random & non-random experiments
7. Write down the sample spaces for following experiment. Also state the nature (type) of the sample spaces.
 - i) A sample is taken to predict the result of the particular assembly poll.
 - ii) A three digit number is formed from the 5 distinct numbers, using each digit only once.
8. Let A, B & C be three events on a sample space Ω
Using the union, intersection symbols, write expressions for the following events
 - i) Only A occurs
 - ii) Exactly one event occurs
 - iii) Exactly two events occur
 - iv) A & B occur but not C
9. Distinguish clearly between deterministic & non- deterministic models. Give suitable examples.
10. List the elements of each of the following sample spaces
 - i) The set of integers between 1 & 50 divisible by 7.
 - ii) The set $\Omega = \{ x / x^2 + 5x - 14 = 0 \}$
 - iii) The set of outcomes when a coin is tossed until a tail or three heads appear.
 - iv) The set $\Omega = \{ x / 2x + 4 \geq 0 \text{ and } x < -2 \}$
- 11) From amongst 2000 literate individuals of a city, 60% read the newspaper A, 55% read B & 20% read neither A nor B. How many individuals read both the newspapers?.

Questions for 6 Marks

1. Let $\Omega = \{1,2,\dots,10\}$, $A = \{1,3,5,7,9\}$, $B = \{1,2,3,4\}$, $C = \{3,4,5,6,7\}$
List the elements of following events
 - i) $(A \cap B) \cup C$
 - ii) $(A \cup B) \cap C$
 - iii) $A \cap B'$
 - iv) $[A \cap (B \cap C)]'$
 - v) $(A \cup B \cup C)'$
 - vi) $(A \cap B) \cup (A \cap C)$
2. Explain the ideas of union & intersection of two events of sample space. Draw appropriate Venn diagram.
3. Give an example of random phenomenon that would be used by
 - i) a teacher
 - ii) a medical researcher
 - iii) an economist
4. Define sample space. Explain various types of sample spaces giving examples of each.
5. Explain i) Occurrence of an event ii) favorable event

6. Explain with one illustration
 - i) Complement of an event
 - ii) Mutually exclusive events
 - iii) Exhaustive events
7. Three coins are tossed together. Let A: exactly 2 coins show heads & B: At least 2 coins show heads. List the elements of A, B, A' & B'. Verify whether A & B are mutually exclusive.
8. Two dice are rolled. Let A be the event that the sum of the points on the uppermost faces is odd & B the event that there is at least one 3 shown.
Describe the events- i) $A \cup B$ ii) $A \cap B$ III) $(A \cap B') \cup A'$
List the elements contained in it.
9. A party worker aspires for a seat to the state Assembly. So he decides to keep on trying for a party ticket until he gets one. Give the sample space for his number of attempts. Also give subsets to represent the following events.
 - i) He needs 2 attempts to get the nomination.
 - ii) He needs not more than 5 attempts to get the nomination

Chapter-2. Probability

A) Questions for 2 marks

1. Give the classical definition of probability.
2. Give the relative frequency approach of probability.
3. Define equiprobable sample space.
4. Define probability of an event.
5. Show that, for any event A, $P(A')=1-P(A)$
6. Prove that the probability of an impossible event is zero.
7. Two cards are drawn from a well shuffled pack of playing cards. Find the probability that both cards are of club.
- 8) Two cards are drawn from a well shuffled pack of playing cards. Find the probability that one is of heart & the other is diamond.
9. Suppose a pair of fair dice is thrown. Find the probability that the sum of the points on the uppermost faces is 7.
10. Suppose a pair of fair dice is thrown. Find the probability that the sum of the points on the uppermost faces is more than 10.

B) Questions for 4 marks

1. Give the classical definition of probability & comment on it.
2. What are the limitations of classical probability?
3. State the axioms of probability.
4. Define i) equiprobable sample space ii) probability of an event
5. For any event A defined on Ω , show that $0 \leq P(A) \leq 1$
6. A & B are two events defined on Ω such that $A \subseteq B$, then show that $P(A) \leq P(B)$
7. State & prove addition theorem of probability for two events
8. Show that the probability that exactly one of the two events A & B defined on Ω will happen is $P(A) + P(B) - 2P(A \cap B)$
9. Arrange the following probabilities in ascending order of magnitude, where A & B are any two events on Ω .
 $P(A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A) + P(B)$
10. Consider a pack of playing cards without face cards. 4 cards from this pack are drawn at random. Find the probability that they belong to
i) different suits ii) different suits & different denomination
11. Which of the following is true. Justify
i) $P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.8$
ii) $P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.3$

C) Questions for 6 marks

- 1) With usual notations, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- 2) For any two events A & B defined on Ω , show that $P(A \cup B) \leq P(A) + P(B)$
- 3) State & prove addition theorem of probability for three events
- 4) If A_1, A_2, \dots, A_n are events defined on Ω then show that $P(\cup A_i) \leq \sum P(A_i)$
- 5) Let A, B, C be any three events defined on Ω , such that
 $P(A) = 3/8$, $P(B) = P(C) = 1/4$ $P(B \cap C) = 0$,
 $P(A \cap B) = 1/8 = P(A \cap C)$
Evaluate i) $P(A \cup B \cup C)$ ii) $P(A \cup C)$ iii) $P(A' \cap B' \cap C')$
- 6) Let A & B be two events on Ω , such that $P(A) = 3/4$ & $P(B) = 5/8$
Prove that i) $P(A \cup B) \geq 3/4$
ii) $3/8 \leq P(A \cap B) \leq 5/8$
iii) $1/8 \leq P(A \cap B) \leq 3/8$

Chapter-3 Conditional Probability And Independence

A) Questions for 2 marks

Chose the correct alternative (Q No. 1 to 10)

- A number is selected at random from 1,2,3. Define $A = \{1,2\}$, $B = \{2,3\}$ $C = \{3\}$ then -----
 - A, B, C are mutually independent events.
 - A, B, C are pair wise but not mutually independent events.
 - A, B, C are mutually independent but not pair wise independent events.
 - None of the above
- Urns I and II respectively contain 5 white, 5 black and 2 white, 8 black balls. An urn is selected at random and a ball is drawn at random from the selected urn. If the ball drawn is white, then probably that the selected urn was II is given by
 - 0.5
 - . = 0,
 - = 1/7
 - = 2/7
- If A_1, A_2, \dots, A_n form a partition of Ω then
 - $\bigcap_1^n A_i = \Omega$, $A_i \cap A_j = \Phi$, $i = j$
 - $\bigcup_1^n A_i = \Omega$, $A_i \cap A_j = \Phi$, $i \neq j$
 - $\bigcap_1^n A_i = \Omega$, $A_i \cup A_j = \Phi$, $i \neq j$
 - $\bigcap_1^n A_i = \Omega$, $A_i \cap A_j = \Phi$, $i \neq j$
- Which of the following statement is true ?
 - A and A^1 form partition of Ω
 - A and Ω form partition of Ω
 - A and A^1 do not form partition of Ω
 - Only two events can not form partition of Ω
- For two events A and B if $P(A) = P(A/B) = 0.25$ and $P(B/A) = 0.5$, then -----
 - A & B are mutually exclusive events.
 - A and B are independent
 - $A \subset B$,
 - $P(A^1 / B) \neq 0.75$
- If A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.25$ then $P(A \cup B)$ is -----
 - 0.65,
 - 0.65
 - 0.1
 - information is not enough.
- If A and B are independent events with $P(A) = 0.4$, $P(B) = 0.5$ then $P(A^1 \cap B) =$ --
 - 0.03
 - 0.9
 - 0.1
 - 0.3

8. Suppose you pass certain examination with probability 0.6 and your friend passes the examination with probability 0.7. Both of you share the same bench and get the same education, hence the events are not independent. What is the probability that both of you pass the examination ?
- 0.42
 - Can't determine but must be ≤ 0.6 .
 - Can't determine but must be ≤ 0.7
 - 1, as you both guarantee to study hard.
9. Which of the following statement is true ?
- $P(A/B) \geq P(A)$ b. $P(A/B) \leq P(A)$ c. $P(A/B) = P(A)$
 - nothing can be said about magnitudes of $P(A)$ and $P(A/B)$.
10. Given that a student has scored distinction in X^{th} standard, probability that he will score distinction in XII^{th} is 0.8. Suppose that he scores distinction in X^{th} is 0.4. What is the probability that he will score distinction in both X^{th} & XII^{th} standard?
- 0.12 b. 0.32 c. 0.032 d. 0.40
- State whether the following statements are true or false. Give justification (Q no. 11 to 15)
11. Independence \Rightarrow mutual exclusiveness.
12. If A^1 & B^1 are independent then A & B independent.
13. Bayes' theorem is used to calculate posterior probabilities of events.
14. If $B \subset A$, then $P(A/B) = 1$
15. If A_1, A_2, \dots, A_n form a partition of Ω then $A_1 \cap B, A_2 \cap B, \dots, A_n \cap B$ form a partition of B .

B) Questions for 4 marks

- Define independence of two events A and B on Ω . Give an illustration.
- Does independence of two events imply that the events are mutually exclusive? Justify.
- Does mutually exclusiveness of two events imply independence? Justify.
- Given that A, B and C are three events defined on Ω then define
 - Pair wise independence
 - mutual independence of A, B and C .
- Prove or disprove that
 - Mutual independence of 3 events \Rightarrow pair wise independence
 - Pair wise independence \Rightarrow mutual independence
- State and prove multiplication theorem for two events A and B defined on a sample space Ω .
- For two events A and B defined on sample space Ω , prove or disprove $P(A/B) = P(B/A)$.
- Let A_1, A_2, A_3 completely independent events defined on Ω .

If $P(A_i) = (1/2)^i$ $i = 1, 2, 3$ find $P(\cup^n A_i)$

9. There are 4 boys and 2 girls in Room No.1 and 5 boys and 3 girls in Room No.2. A girl from one of the rooms laughed loudly. What is the probability that the girl who laughed loudly was from Room No.2?
10. The probability that a man will be alive 20 years hence is 0.3 and probability that his wife will be alive 20 years hence is 0.4. Find the probability that after 20 years.
 - (i) Both will be alive
 - (ii) Only the man will be alive
 - (iii) Only the wife will be alive and
 - (iv) At least one of them will be alive.
11. An article manufactured by a company consists of two parts A and B. In the manufacturing process of part A, 9 out of 100 are likely to be defective; similarly 5 out of 100 are likely to be defective in the process of part B. Calculate the probability that the assembled parts will not be defective.
12. In a certain school, examination results showed that 20% students failed in Mathematics, 5% failed in English while 10% failed in both Mathematics and English. Are the two events failing in Mathematics and failing in English independent. ?
13. A town has 3 doctors A, B and C operating independently. Then probability that doctor A available is 0.9 and that for doctor B is 0.6, for doctor C is 0.7; what is the probability that at least one doctor is available when needed ?
14. There are two sections I and II in Statistics paper. The probability that a candidate passes in Section – I is 0.6 and that he passes in Section-II is 0.7. What is the probability that a particular candidate passes only in any one of the two sections?
15. Two urns identical in appearance contain respectively 3 white and 2 black balls, and 2 white and 5 black-balls. One urn is selected at random and a ball is drawn from it. What is the probability that it is black?
16. A, B, C form a partition of Ω . If $3 P(A) = 2 P(B) = P(C)$ find $P(A \cup B)$
17. A and B are two events defined on a sample space Ω state the nature of relationship between A and B in each of the following :
 - (a) $P(A/B) = 0$
 - (b) $P(A/B) = P(A)$.
18. If A and B are two events on a sample space Ω . Prove with usual notation.
 $P(A) = P(A/B) \cdot P(B) + P(A/B^1) \cdot P(B^1)$
19. If A and B are mutually exclusive non empty events of Ω , show that
 $P(A/A \cup B) = P(A) / (P(A) + P(B))$
20. Given that $P(A_1) = P(A_2) = P(A_3) = 1/3$ and $P(B/A_1) = 2/7$,

$P(B/A_2) = 4/9$, $P(B/A_3) = 1/5$, find $P(A_1/B)$.

C) Questions for 6 marks

- If A and B are two independent events defined on Ω ; then prove that.
 - A and B^c are independent,
 - A^c and B are independent
 - A^c and B^c are independent
- Show that conditional probability satisfies all axioms of probability.
- If A, B, C are any three events defined on Ω with $P(B) > 0$, then prove that $P(A \cup C/B) = P(A/B) + P(C/B) - P(A \cap C/B)$
- Define partition of a sample space. State and prove Baye's theorem.
- If A_1, A_2, \dots, A_n are partition of a sample space and if B is any event defined on Ω , prove that $P(B) = \sum_{j=1}^n P(A_j) P(B/A_j)$
- Of the three events A, B and C. A and B are mutually exclusive, A and C are independent. B and C are independent. If $P(A) = 1/4$, $P(B) = 1/3$, $P(C) = 1/6$. Find
 - $P(A \cup B)$
 - $P(A \cap C)$
 - $P(A \cup B \cup C)$.
- A husband and wife appear for two vacancies in the same post. The probability of husband's selection is $1/7$ and that of wife's selection is $1/5$. What is the probability that-
 - Both of them will be selected?
 - Only one of them will be selected?
 - None of them will be selected?
- A bag contains either 40 percent red balls or 60 percent red balls. Both of these possibilities are equally likely. A ball is drawn at random and found to be red ball. What is the probability that the bag has 60 percent red balls? If second ball is drawn and found to be red, what is the new probability that the bag contains 60 percent red balls?
- In a typing – pool, three typists share the total work in the ratio of 30 percent, 35 percent and 35 percent of the work done by them. The first, second and the third typist spoil the work to the extent of 3 percent, 4 percent and 5 percent respectively. A completed work is selected and found to be spoiled. What is the probability that the work was done by the third typist?
- In a bolt factory, three machines A, B, C produce respectively 25%, 35% and 40% of the days production. Of the total of their output 5%, 4% & 2% are defective bolts respectively from A, B and C. A single bolt is drawn at random from the days' production and is found defective. Determine the probabilities that it was manufactured by machines A, B and C.

11. A recently conducted study indicates that 60 percent of commerce students tend to use fantasies as a mechanism for overcoming frustration caused by solving business statistics problems. While the remaining do not turn to fantasies for this reason. A psychologist has developed a test for measuring whether the student fantasies or not. However the test is not yet full-proof and the test yields a positive result for 60 percent of the students who fantasize and yields positive result for 20 percent for non-fantasiers. Given these details, determine the posterior probability that a randomly selected student is a fantasizer, given a positive test result.
12. A firm has four plants producing the same item at all plants. The first plant produces 30% of the total production, second plant 25%, third plant 35% and the fourth plant 10%. The firm has a single warehouse to store. From the past records it has been found that 5%, 10%, 15% and 20% of the items produced at plants 1, 2,3 & 4 respectively are defectives. Before the shipment of the items to a dealer, one unit is selected and found defective. What is the probability that the item was produced in plant 3?
13. Bag I contains 3 blue and 4 red balls. Bag II contains 2 blue and 6 red balls. Bag III contains 1 blue and 8 red balls. A bag is chosen at random; a ball is drawn randomly from the bag. It turns out to be blue. Find the probability that bag – I was chosen.
14. An explosion in a factory manufacturing explosives can occur due to (i) short circuit (ii) defects in machinery (iii) negligence of workers and (iv) sabotage. The probabilities of these four causes are known to be 0.3, 0.2, 0.4 and 0.1 respectively. The engineers feel that an explosion can occur with probability
 - (i) 0.3 if there is a short circuit.
 - (ii) 0.2 if there are defects in machinery
 - (iii) 0.25 if the workers are negligent and 0.8 if there is sabotage.
 Given that an explosion has occurred, determine the most likely cause of it.
15. Consider that there are three identical bags, A, B and C. The bag A contains 2 gold coins bag B contains 2 silver coins and bag C contains 1 silver and 1 gold coin. What is the probability of selecting bag A out of the three given that a gold coin is selected?

Chapter 4. Univariate Probability Distributions

A) Questions for 2 marks

1. State, giving reasons, whether the following statements are true or false.
 - i) A discrete random variable (d.r.v.) assumes only finite number of values.

- ii) A d.r.v. is defined on a discrete sample space.
- iii) A d.r.v. takes only positive integral values.
- iv) Sum of probabilities of all values of a r.v. is less than one.
- v) The p.m.f. of a d.r.v. is a non decreasing function.
- vi) We can not find probability of interval for discrete variable.
- vii) A distribution function is an increasing function.
- viii) The graph of distribution function of a d.r.v. contains of number of steps.
- ix) Medium M of a d.r.v. is that value of the r.v. for which $F(M) = \frac{1}{2}$

2. State the r.v. and its range set in each of the following cases.

i) A pair of fair dice is rolled and

- a) Sum of the two numbers is noted
- b) Difference between the two numbers noted
- c) Maximum of two numbers
- d) Minimum of two numbers.
- e) Absolute difference between the two numbers
- f) Product of the two numbers

ii) Four unbiased coins are tossed simultaneously and numbers of runs are observed.

(A run is an unbroken chain of same symbol.)

iii) A bag contain three red and four black balls .Three ball are drawn at random without replacement and number of red balls are recorded.

iv) A random sample of Five mangoes is taken from a dozen of mangoes which contain 3 bad mangoes number of good mangoes in the samples is recorded.

3. Determine k such that the following functions are p.m.f

i) $f(x) = kx \quad x = 1, 2, 3, \dots, 10.$

ii) $f(x) = \frac{k 2^x}{x!} \quad x = 0, 1, 2, 3.$

iii) $f(x) = k(2x^2 + 3x + 1), x = 0, 1, 2, 3.$

iv)

X	0	1	2	3	4
P(X=x)	k	3k	5k	2k	k

v)

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

vi)

X	0	1	2
1. $P(X=x)$	2. $3k^3$	3. $4k-10k^2$	4. $5k-1$

vii) $f(x)=k(x^5)$, $x =0,1,2,3,4,5$.

B) Questions for 4 marks

1. Verify whether the following can be looked upon as p.m.f. for the given values of x.

i) $p(x)=1/4$ $x=0,1,2,3$.

ii) $p(x)=(x+1)/10$ $x=0,1,2,3$.

iii) $p(x)=x^2/30$ $x=0,1,2,3,4$.

iv) $p(x)=(x-2)/5$ $x=1,2,3,4,5$.

v) $p(x) = x^2 /14$ $x=1,2,3$.

vi) $p(x) = (x-1)/2$ $x=0,1,2$.

2. Define cumulative distribution function (c.d.f.) of a discrete r.v. and state its important properties.

3. Define median of discrete probability distribution.

4. Define mode of discrete probability distribution.

C) Questions for 6 marks

1. Given the following c.d.f. $F(x)$ of d.r.v. x obtain the p.m.f. of x also obtain the median & mode of x .

i) $F(x)=0$ $x<0$
 $=0.2$ $0 \leq x < 2$
 $=0.5$ $2 \leq x < 4$
 $=0.7$ $4 \leq x < 6$
 $=0.8$ $6 \leq x < 8$
 $=1$ $x \geq 8$

ii) $F(x)=0$ $x < 1/4$
 $=1/8$ $1/4 \leq x < 1/2$
 $=1/4$ $1/2 \leq x < 3/4$

$$\begin{aligned}
&=1/2 && 3/4 \leq x < 1 \\
&=3/4 && 1 \leq x < 5/4 \\
&=1 && 5/4 \leq x < 3/4 \\
\text{iii) } F(x) &= 0 && \text{if } x < 5 \\
&=1/8 && 5 \leq x < 10 \\
&=7/24 && 10 \leq x < 15 \\
&=2/3 && 15 \leq x < 20 \\
&=11/12 && 20 \leq x < 25 \\
&=1 && x \geq 25.
\end{aligned}$$

2. A d.r.v. assumes value -2.5, -1.5, 0.5, 1.5 and 2.5 if $P(x=-2.5)=P(x=-1.5)$

$$P(x=1.5)=P(x=2.5), P(x<0.5)=P(x>0.5)=P(x=0.5)$$

Obtain (i) p.m.f. of x.

(ii) The distribution function of x.

3. A r.v. assumes values 1, 2, 3, 4, 5 such that $P(x=1) = P(x=2)$; $P(x=4) = P(x=5)$;

$$P(x<3) = P(x=3) = P(x>3) \text{ write down the p.m.f. of } X \text{ \& evaluate } P(X \leq 3).$$

4. A r.v. x takes values 0, 1, 2, 3, 4 such that $P(1 < x \leq 4) = 0.55$; $P(x \leq 1) = 0.25$; $P(x=2) = 2P(x=1)$ and

$$P(0 < x \leq 2) = 0.45; \text{ Find the probability distribution of } x.$$

5. A d.r.v. x assumes values -3, -2, -1, 0, 1, 2, 3 such that ,

$$P(x=-3)=P(x=-2)=P(x=-1);$$

$$P(x=1)=P(x=2)=P(x=3);$$

$$P(x<0)=P(x=0)=P(x>0);$$

Obtain the probability function & distribution function of x. Hence Find the Probability distribution function of $Y=3x+4$.Also find the median of x.

6. A r.v. x has the following distribution

X	0	1	2	3
P(X)	1/5	2/5	1/5	1/5

Find the Probability distribution of

i) $w = x-1$

ii) $y = (3x+2)/2$

iii) $z = x^2+2$

7. Let a d.r.v. assumes values 5, 1, 0, 15, 20 with equal probability distribution of $y=(x-5)/5$

Also determine the median of Y. Can you determine the mode of Y? Explain.

Chapter -5: Mathematical Expectation

A) Questions for 2 marks

1. Define variance of a discrete random variable
2. Define expectation of a r.v.
3. Show that $E(k)=k$
4. The mean and variance of scores (x) in a certain test are 120 and 100 respectively. Find the

$$\text{mean \& variance of } y = \frac{X - 120}{10}$$

5. Prove that variance of a constant zero

B) Questions for 4 marks

1. Show that variance is invariant to change of origin but not of scale.
2. A random variable X assumes 7 values $-3, -2, -1, 0, 1, 2, 3$ with equal probability find $E(X)$ and $E(X^2)$.
3. Show that $\text{Var}(aX+b)=a^2 \text{Var}(X)$, where a and b are constant
4. With usual notations, prove that $E(X-k)^2 = \text{Var}(X)+[E(X)-k]^2$
5. A r.v X takes values $0, 1, 2, \dots, n$ with probabilities proportional to the binomial coefficients $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n}$ find $E(X)$.
6. The probability mass function of a r.v. X is

$$p(x) = \frac{1}{10} \quad ; x=21, 22, 23, \dots, 30$$
$$= 0 \quad ; \text{otherwise}$$

find $E(X)$.

7. Prove that $E(X^2) \geq [E(X)]^2$
8. The probabilities that a man fishing at a particular place will catch 1, 2, 3, 4 fish are 0.4, 0.3, 0.15, 0.15 resp.. What is the expected no. of fish caught?
9. Two cards are drawn at random from a box which contains five cards numbered 1, 1, 2, 2 and 3. Let X denotes the sum of the numbers. Find the expected value of the sum.
10. A man wishes to open the door of his house in dark. He has a bunch of n keys out of which only one key works. He tries the keys one by one, removing the unsuccessful keys. Find the expected number of keys he tries before the door is opened.
11. A baker man sells 5 types of cakes. Profit due to sale of each type of cake is respectively Rs. 1, 1.5, 0.5, 0.75 and 0.25. The demands for these cakes are 10%, 5%, 20%, 50% and 15% respectively. What is the expected profit per cake?

12. A player tosses two fair coins. He wins Rs. 5 if 2 heads appear, Rs. 2 if 1 head appears and Re. 1 if no head appears. Find his expected amount of winning and variance of winning.
13. For a discrete r.v. X , $E(X) = 10$ and $\text{Var}(X) = 25$. Find the positive values of a and b such that $Y = aX - b$ has mean 0 and variance 1.
14. Show that $E(ax+b) = aE(x) + b$

C) Questions for 6 marks

1. A discrete random variable x take value $1, 2, \dots, n$ with equal probability $1/n$. find the mean and variance of x if the ratio of $\text{var}(X)$ to $E(X)$ is equal to 4 find the value of n . What will be the value of n if $\text{var}(X) = E(X)$?
2. A r.v. X has following probability mass function

X	- 2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	k

Find value of k & calculate mean and variance of x .

3. Let X be a r.v. with following as the p.m.f. :-

X	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

Find $E(X)$ and $\text{Var}(X)$.

4. Let X be discrete r.v. with p.m.f.

$$P(X=x) = \frac{x}{15} \quad \text{for } x=1,2,3,4,5$$

$$= 0 \quad \text{otherwise.}$$

Find $E(X)$ & $\text{Var}(2x-3)$.

Chapter-6 Bivariate Probability Distributions

A) Questions for 2 marks

1. The joint p.m.f. of (X, Y) is

$$P(x, y) = \frac{2x+3y}{72} \quad x = 0, 1, 2, \quad y = 1, 2, 3$$

$$= 0 \quad \text{otherwise}$$

Find conditional p.m.f. of Y given $X = 1$.

- Define a bivariate discrete random variable .
- Define Joint probability mass function of (X,Y).
- Show that when X and Y independent , the conditional distribution of X given Y = yj, is the marginal distribution of X.
- When are we say that two discrete random variables independent ?

B) Questions for 4 marks

- Give two practical situations where use of bivariate random variables in needed.
- Explain what is meant by marginal probability distributions of X and Y.
- Explain what you understand by (i)Conditional distribution of X given Y = yj
(ii) Conditional distribution of Y given X = xj .
- Show that the conditional probability mass function of X given Y and conditional probability mass function of Y given X = xj satisfy the conditions of probability mass function.
- Show that when X and Y independent , the conditional distribution of X given Y = yj, is the marginal distribution of X.
- Define the joint distribution function of a two dimensional discrete random variable. Also state its properties.
- Let a fair coin be tossed three times. If X and Y denote the number of heads and the number of runs respectively, obtain the joint probability distribution of X and Y.
- A bag contains 2 white , 3 red and 5 green marbles. Three marbles are drawn at random from this bag without replacement .Obtain the joint probability distribution of the number of white marbles (X) and the number of red marbles (Y) so drawn.
- Below is given the joint probability distribution of X and Y

Y	1	2	3
X			
1	5c	c	3c
2	4c	3c	4c
3	2c	3c	2c

- (i)Find the value of c (ii) Find the conditional distribution of X given Y = 3. Also find the conditional distribution of Y given X = 1.

10. The joint of (X, Y) is given by

$$P(x,y) = c (x^2 + y^2) ; \quad c > 0 \quad x = -1 , 1 \quad y = -2 , 2$$

$$= 0 \quad \text{otherwise}$$

Obtain (i) c

(ii) the marginal p.m. f of X.

(iii) Are X and Y independent?

11. Two beads are selected at random without replacement from a bowl containing 4 blue, 1 red and 2 black beads. Let X denote the number of red beads drawn, Y denote the number of black beads drawn. Find the joint p.m.f. of (X, Y).

C) Questions for 6 marks

1. The joint probability distribution of a two dimensional random variable (X, Y) is given below

Y \ X	- 1	0	1	2	Total
-2	3k	0	k	3k	8k
1	6k	0	3k	3k	12k
2	0	0	3k	4k	7k
Total	9k	k	7k	10k	1.0

Find (i) the value of k. (ii) $P(x < 0 , Y \geq 0)$

(iii) $P(x = Y)$ (iv) $P(x^2 + Y^2 \leq 2)$.

2. Below is given the joint probability distribution of X and Y

Y \ X	1	2	3
1	5c	c	3c
2	4c	3c	4c
3	2c	3c	2c

(i) Find the value of c.

(ii) Find $P(X \leq 2, Y \leq 3)$.

(iii) Find $P(X \leq 2, Y > 2)$.

(iv) Find the marginal probability distribution of X and Y.

(v) Are X and Y independent ?

3. The joint probability distribution of X and Y is given by

$$P(x,y) = c(2x + 5y), \quad c > 0 \quad X = 1,2 \quad Y = 1,2$$

$$= 0 \quad \text{otherwise}$$

Find (i) The marginal probability mass functions of X and Y.

(ii) The conditional probability mass function of X given $Y = 2$

4. Let the joint probability mass function of X and Y be

$$P(x,y) = \begin{cases} \frac{x+y}{21}, & x = 1,2,3 \\ & y = 1,2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Are X and Y independent ?

(ii) Find $P(XY \text{ is odd})$.

(iii) Find the conditional probability distribution of X given $Y = 2$.

5. Following is the joint probability distribution of (X, Y)

	Y	1	2	3	4
X					
0		1/24	1/12	1/12	1/24
1		1/12	1/6	1/6	1/12
2		1/24	1/12	1/12	1/24

(i) Obtain conditional probability distribution of X given $Y = 2$

(ii) Calculate $P(X \leq 1 | Y = 1)$

(iii) Calculate $P(Y \geq 3 | X \geq 1)$

(iv) Calculate $F(1, 1)$.

6. Let (X, Y) be a discrete random variable with the following joint

Probability mass function.

Y	0	1	2	3
X				
0	k	3k	2k	4k
1	2k	6k	4k	8k
2	3k	9k	6k	12k

(i) Find k .

(ii) Are X and Y independent ?

(iii) Calculate $P(X \leq Y, Y \geq 1)$ and $P(X = Y)$.

Chapter- 7 Expectation of functions of two dimensional r.v.

A) Questions for 2 marks

1. Give two practical situations where use of bivariate r.v. is needed
2. Define a Expectation of function of bi-variate discrete r.v.
3. Define conditional expectation of $g(x)$ given $Y=y$
4. Define conditional expectation of x given $Y=y$
5. Define conditional expectation of $g(y)$ given $Y=y$
6. Define conditional expectation of Y given $Y=y$
7. Explain conditional Mean
8. Define conditional Variance X given $Y=y$
9. Define conditional Variance Y given $X=x$
10. Define covariance of a r.v.s.
11. Define correlation coefficient between X and Y
12. Give any four properties of co-variance
13. Give any four properties of correlation coefficient
14. Interpret the value of correlation coefficient?
15. Explain independence implies uncorrelation
16. Define raw moments in bivariate discrete r.v.
17. Define Central moments of bi-variate discrete r.v.
18. A r.v. (X,Y) has joint pmf as follow

	X	0	1	2
Y				
	-1	0.1	0.2	0.3
	+1	0.1	0.1	0.2

What is conditional expectation of X given $Y= y$

19. Write formula for variance of linear combination of X and Y
20. Prove that $\text{Cov}(X,X) = V(X)$
21. Is it true $\text{Cov}(X, -Y) = -\text{Cov}(X,Y)$
22. Is it true that $\text{Cov}(-X, -Y) = \text{Cov}(X,Y)$

B) Questions for 4 Marks

1. With usual notation show that $E(X+Y) = E(X) + E(Y)$
2. With usual notation show that $E(XY) = E(X)*E(Y)$, If X and Y are independent
3. Give example of joint probability distribution of (X, Y) , where $E(X*Y) = E(X)*E(Y)$ holds but X & Y are not independent.
4. Define covariance between two r.vs, what does it measure?

5. Does $\text{Cov}(X, Y) = 0$ imply that X and Y are independent? Justify
6. Prove that $\text{Cov}(aX, bY) = a.b.\text{Cov}(X, Y)$ where a and b are constant
7. Prove that $\text{Cov}(X+c, Y+d) = \text{Cov}(X, Y)$ Where c & d are constants.
8. Suppose X & Y are two discrete rvs, prove that $V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab.\text{Cov}(X, Y)$
9. Suppose X and Y are two discrete r.v.s
Prove that $V(aX - bY) = a^2V(X) + b^2V(Y) - 2ab.\text{Cov}(X, Y)$
10. For X and Y two discrete r.v.s,
Prove that $V(X + Y) = V(X) + V(Y) - 2\text{Cov}(X, Y)$
11. For X and Y two independent discrete r.v.s, prove that $V(X + Y) = V(X) + V(Y)$
12. For X and Y two independent discrete r.v.s, prove that $V(X - Y) = V(X) + V(Y)$
13. Define correlation coefficient between two discrete r.v X and Y . Give the interpretation of the various values of correlation coefficient.
14. Prove in case of two discrete r.vs independence \Rightarrow Uncorrelatedness
15. Prove in case of two discrete r.vs Uncorrelatedness does not imply independence
16. Define conditional expectation of X given $Y=y_i$ and conditional expectation of Y given $X=x_i$. Also give their interpretation
17. Define conditional Variance of X given $Y=y_i$ & Variance of Y given $X = x_i$
18. Define (r, s) th raw moment (μ'_{rs}) of discrete bi-variate distribution of (X, Y)
19. Define (r, s) th central moment μ_{rs} of (X, Y)
20. With usual notation show that $\mu'_{10} = E(X)$, $\mu'_{01} = E(Y)$ & $\mu'_{11} = E(XY)$
21. With usual notation show that $\mu_{20} = V(X)$; $\mu_{02} = V(Y)$.
22. With usual notation show that $\text{Corr}(X, Y) = \mu_{11} / \sqrt{\mu_{20} * \mu_{02}}$
23. X and Y are two independent discrete rv.s with $\sigma_x = 3$ and $(2X + 3Y) = 72$ compute σ_Y
24. Suppose X_1, X_2, X_3 are three discrete r.v.s. with means 2, 4 and 5
find i) $E(2X_1 + X_2 + X_3)$ ii) $E(X_1 - X_2 - X_3)$
25. Suppose X_1, X_2, X_3 are three discrete r.v.s. with sds 1, 2, 3 respectively find SD of $(X_1 + X_2 + X_3)/3$
26. Prove that $V(X - Y) = V(X) + V(Y) - 2\text{Cov}(X, Y)$
27. Prove that $\text{COV}(X, X+Y) = V(X) + \text{COV}(X, Y)$
28. For (X, Y) , a bivariate discrete rv $\sigma_X = 9$, $\sigma_Y = 4$, $\text{Cov}(X, Y) = 4$ find $V(2X - 3Y)$
29. For (X, Y) a bi-variate discrete rv such that $\text{Cov}(X, Y) = 4$
Find i) $\text{Cov}(2X, 3Y)$ ii) $\text{Cov}(X+5, 2Y - 6)$
30. For the following probability distribution

Y \ X	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Find $E(2X - Y)$

31. Let (X, Y) have the following joint pmf

Y \ X	-1	0	1	
0	0	1/6	0	1/6
1	1/6	0	1/6	1/3
2	0	1/2	0	1/2
P.J	1/6	2/3	1/6	

Show that $\text{COV}(X, Y) = 0$

32. For two discrete rvs X and Y , $V(X) = V(Y) = 1$, $\text{Cov}(X, Y) = 1/2$. find $V(3X - 4Y)$

33. For two discrete rvs X and Y , $V(X) = V(Y) = 1$ $\text{Cov}(X, Y) = 1/2$.
find $\text{Corr}\{(X+5)/2, (Y-6)/3\}$

C) Questions for 6 Marks

1. With usual notation prove that

- a) $E(X+Y) = E(X) + E(Y)$
- b) $E(XY) = E(X) * E(Y)$ if X and Y are independent

2. Give an example of joint probability distribution of (X, Y) where $E(XY) = E(X) * E(Y)$ holds but X & Y are not independent.

3. If X_1, X_2, \dots, X_n are independent discrete rvs with common variance σ^2 , show that

$$\text{Var}(\bar{X}) = \sigma^2/n \quad \text{where } \bar{X} = \sum x_i / n$$

4. Obtain $E(2X - Y)$ and $E(XY)$ for the following probability distribution

Y \ X	0	1	2
1	0.06	0.15	0.09
2	0.14	0.35	0.21

5. Obtain Conditional mean and Variance of X given Y=3 for the following joint probability distribution

	Y	1	2	3
X				
	0	0.1	0.2	0.3
	1	0.1	0.1	0.2

6. The joint pmf of X and Y is

	Y	0	1	2
X				
	-1	1/6	0	1/2
	+1	1/4	1/3	1/6

Find $E(Y/X=1)$ and $V(Y/X=1)$

For Q:7 to 14

If X and Y are two discrete rvs with joint pmf

$$P(X, Y) = 1/4 \text{ for } X = 0, 1, Y = 0, 1$$

$$= 0 \text{ O.W.}$$

7. Calculate Cov (X, Y)
- 1) Calculate V (X+Y)
 - 2) Are X and Y independent
 - 3) Find V (3X – 4Y)
 - 4) Find r (X, Y)
 - 5) Find r (2X, 3Y)
 - 6) $E\{X/(Y=1)\}, V\{X/(Y=1)\}$
 - 7) $E\{Y/(X=0)\}, V\{Y/(X=0)\}$

Chapter- 8 Discrete Uniform Probability Distribution

A) Questions for 2 marks

1. Give two real life situations where uniform distribution can be realised
2. Find the mean of uniform distribution
3. State which distribution can be applicable in following situation
 - a) the birth day of a person it may be either Sunday , Monday - - - Saturday
 - b) a computer generates a digit randomly

4. The p.m.f. of uniform random variable X is $p(x) = k$ for $x=5,6,7,\dots,16$, $k.>0$
Find the value of k.

B) Questions for 4 Marks

1. Define discrete uniform probability distribution & obtain its mean and variance
2. For a uniform distribution with p.m.f.

$$P(X) = 1/n, X = 1,2,..n$$

$$= 0 \text{ otherwise}$$
 Find $E(X/n+1)$ and $\text{var}(X/n^2 - 1)$

C) Questions for 6 Marks

1. The p.m.f. of uniform random variable X is $p(x) = k$ for $x=5,6,7,\dots,16$, $k.>0$
Find the value of k. Find further probability that
 - (i) the value of X greater than 8,
 - (ii) the value of X greater than 6 but less than or equal to 14
 - (Iii) the value of X greater than than or equal to 10
2. A special die has (n+1) faces marked with numbers $0,1,2,\dots,n$. Assuming that the die is unbiased, find the probability distribution of the number on the top face. , find also the probability that the number on the top face is greater than 4 but less than or equal to (n-1) ?
3. Compute γ_1 for a discrete uniform random variable and comment upon its value.
4. Obtain mean and variance of uniform random variable taking values $1,2,\dots,n$.

Chapter 9 Bernoulli Distribution

A) Questions for 2 marks

1. Define Bernoulli distribution.
2. Give three examples of real life situations where Bernoulli distribution can be used.

B) Questions for 4 Marks

1. Obtain mean and Variance of Bernoulli distribution.
2. Find m.g.f. of Bernoulli random variable. Hence show that the sum of n independent & identically distributed r.v.s. follows Binomial distribution.
3. Show that all raw moments of Bernoulli(P) r.v. are equal to 'p'

C) Questions for 6 Marks

1. Obtain mean, Variance and μ_3 for a Bernoulli r.v. when is the distribution symmetric?
2. Show that product of n independent Bernoulli(p) r.v.s. is a Bernoulli r.v. with parameter p^n .
3. For a Bernoulli r.v. X, $\mu_3 = 0.7$ what is $\text{Var}(X)$?
4. A r.v. X follows Bernoulli distribution with $\mu_3 = 0$. what is the value of p?

Chapter - 10 Binomial Distribution

A) Questions for 2 marks

1. Define Binomial distribution.
2. Find mean of binomial distribution
3. Define mode of Binomial distribution
4. Is it possible to have $E(X) = 3$ & $V(X) = 5$? Justify

B) Questions for 4 Marks

1. State and prove additive property of Binomial distribution
2. Prove that under certain conditions to be stated, Binomial distribution is approximated by Poisson distribution.
3. Suppose $X \sim B(n, p)$ I) If $E(X) = 6$ & $\text{Var}(X) = 4.2$ find n & p ii) if $p = 0.6$ $E(X) = 6$, find n & $\text{Var}(X)$
4. Let $X \sim B(n = 8, p = 1/4)$ find I) $P(X=3)$ ii) $P(X \leq 3)$
iii) $P(X \leq 6)$ iv) Mean & Variance of X
5. Let $X \sim B(2, p)$ & $Y \sim B(4, P)$. if $P(X/1) = 5/9$. find $p(Y/1)$
6. A fair coin is tossed 10 times. Find the probability of getting
I) Exactly 6 heads ii) at least 2 heads iii) at most 8 heads
7. Explain how binomial distribution is applied in case of SRSWR.
8. State and prove the recurrence relation for binomial probability? What is its use?
9. Obtain in mode of B (n, p) variable. Is it always unique?
10. If $X \sim B(n, p)$, show that $n - x \sim B(n, 1 - P)$
11. Let X follows Binomial distribution with mean = 10 and Variance = 5.
Find i) $P(X = 5)$ ii) $P(2 < X < 10)$ iii) $P(X \leq 10)$
12. Compute mode of a binomial distribution for I) $n = 10, p = 1/3$
ii) $n = 11, p = 1/2$.
13. For a B (n, p) distribution, $P(X=1) = 0.0768$, $P(X=2) = 0.2304$,

Find the value of p.

C) Questions for 6 Marks

1. Let $X \sim B(n, p)$. with usual notations, prove that

$$\mu_{r+1} = pq \left[\frac{d \mu_r / dp + nr \mu_{r-1}}{r} \right] \quad r=1,2, \dots$$

2. Let $b(k, n, p) = {}^n C_k p^k (1-p)^{n-k}$, $K=0,1,2,\dots,n$

Then prove that

- i) $b(k, n, p) = b(n-k, nq)$
- ii) $b(k, n+1, p) = p b(k-1, n, p) + q b(k, n, p)$
- iii) $\sum b(k, n, p) = 1 - \sum b(k, n, q)$

3. Let X be a binomial random Variable with parameters n & P. Prove that

$$E \left(\frac{X}{n} - p \right)^2 = -\text{Cov} \left(\frac{X}{n}, n - \frac{X}{n} \right).$$

4. Let X & Y be two independent binomial Variates with parameters

(n_1, p) & (n_2, p) respectively. Find the conditional distribution of X given $X+Y = n$

5. Let $X \sim B(n, p)$ I) if $n=15$ & $E(X)=6$ find p & $V(X)$

ii) If $E(X)=9$ & $V(X)=3.6$, find n & p

iii) If $p=0.7$ & $E(X)=14$, find n & $V(X)$

iv) If $n=25$ & $E(X)=10$, find p & $V(X)$.

6. Define Binomial distribution. Give interpretation of a $B(n, p)$ r.v. Also

Mention three real life situations, where the distribution's application

7. Let $X \sim B(n, p)$. Obtain the mean and Variance of X. show that $\text{Var}(X) < E(X)$

8. Compute μ_3, μ_4, μ_1 & μ_2 of $B(n, p)$ Variable. Discuss the skew ness and

kurtosis of the distribution over various values of 'p'.

9. For a $B(n, p)$ distribution, with usual notations, prove that

$$\mu'_{r+1} = pq \left[\frac{d \mu'_r / dp + n/q \mu'_r}{r} \right] \quad r=1,2, \dots$$

10. A student prepares for an examination by studying a set of 10 problems. He can work 7 of the 10. if the professor chooses randomly 5 of the 10 problems for the exam. What is the probability that the student can work at least 4 of them?

11. A radar system has a probability of 0.1 of defecting a certain target during a single scan. Find the probability that the target will be detected

- a) at least twice in four scans
- b) at most once in four scans

12. If the probability that any person 65 years old will be dead within a year is 0.05. Find the prob. that out of a group of 7 such persons

- i) Exactly one
- ii) None
- iii) at least one
- iv) Not more than one
- v) all of them will die within a year.

13. If X and Y are independent binomial Variation with $X \sim B(n, 1/2)$.
Find $P(X+Y=5)$, $P(X+Y \leq 1)$, $P(X+Y/2 \leq 1)$, $P\{3(X+Y) \leq 6\}$
14. Suppose that the prob. That a light in a classroom will be burnt out is $1/3$. The classroom has in all 5 lights and it is unusable. If number of Lights' burning is less than two. What is the probability that the classroom is unusable on random occasion?
15. Let X & Y be two independent binomial variation with parameters $(n_1 = 5, p = 0.4)$ & $(n_2 = 6, p = 0.4)$ respectively.
Find i) $P(X+Y = 4)$ ii) $P(X+Y \leq 8)$
 iii) $P\{X = 1/(X+Y = 7)\}$ iv) $P\{Y = 2 / (X+Y = 8)\}$

Chapter 11 Hyper Geometric Distribution

A) Questions for 2 marks

1. Define Hyper geometric distribution.
2. Mention its two practical applications.
3. How does it differ from Binomial distribution as far as the application is concerned?
4. Obtain its mean

B) Questions for 4 Marks

1. State and prove the theorem which facilitates binomial approximation to a hyper geometric distribution.
2. Show w.y.n. that the r^{th} factorial moments of a H (N, M, n) distribution is given by
$$\mu_{(r)} = \binom{n}{r} M^{(r)} / N^{(r)}$$
3. A parcel of 12 books contains 4 books with loose binding what is the prob. That a random selection of 6 books (without replacement) will contain 3 books with loose binding?

C) Questions for 6 Marks

1. Of the 100 people in a certain village, 40 always tell the truth & remaining always lie. A sample of 10 persons is chosen from these people. What is the prob. That the sample will contain
i) Three ii) No iii) all liers?
2. A group of 20 cricket players contain 7 Maharashtrians & remaining Non-Maharashtrians. An Indian team of 12 players is to be formed
i) What will be the average no. of Maharashtrians selected in the team?

- ii) What is the probability that one fourth of players in the team are Maharashtrians?
3. Seventy percent of the letters received by the popular T.V. program 'Surabhi' are written by ladies. What is the probability that
- i) All the five prizes awarded by Surabhi are bagged by ladies
- ii) Exactly 2 by ladies iii) at most by ladies?
